YEAR 8
KNOWLEDGE ORGANISERS

White Rose Maths

RATIO & PROPORTION...

Ratio

What do I need to be able to do?

- Understand ratio
- Ratio problems (whole given)
- Ratio problems (part given)
- Ratio problems (difference given)
- Simplify ratios
- Express ratios in the form 1:n and n:1 (E)
- Compare ratios and fractions
- Solve problems with ratio

Keywords

Tier 2:

Ratio: a statement of how two numbers compare

Equal Parts:: all parts in the same proportion, or a whole shared equally

Proportion: a statement that links two ratios

Order: to place a number in a determined sequence

Part: a section of a whole

Equivalent: of equal value

Factors: integers that multiply together to get the original value

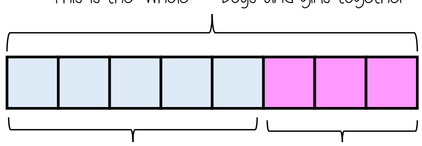
Scale: the comparison of something drawn to its actual size.

Representing a ratio

"For every 5 boys there are 3 girls"

5:3

This is the "whole" – boys and girls together

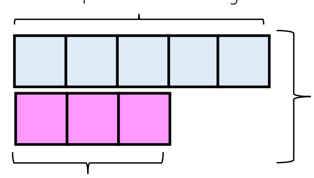


This represents the 5 boys

This represents the 3 girls

Double Number Line

This represents the 5 boys



This is the "whole" – boys and girls together

This represents the 3 girls

Sharing a whole into a given ratio

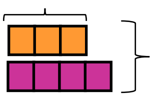
James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy

3 : 4

James



£350

Lucy

Find the value of one part

Whole: £350

7 parts to share between

(3 James, 4 Lucy)

£350 ÷ 7 = £50

□ = one part = £50

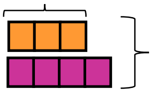
Put back into the question

James: Lucy

× 50 3 : 4 × 50

£150:£200

James = 3 x £50 = £150



£350

Lucy = 4 x £50 = £200

Sharing in a ratio given a difference

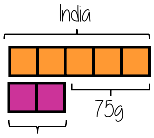
India and Beau share some popcorn in the ratio of 5:2
If India has 75g more popcorn than Beau, what was the original quantity?

Model the Question

India: Beau

5 : 2

India



75g

Beau

Find the value of one part

Difference: 75g

75g ÷ 3 = 25g

□ = one part = 25g

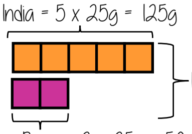
Put back into the question

India: Beau

× 25 5 : 2 × 25

125g : 50g

India = 5 x 25g = 125g



175g

Beau = 2 x 25g = 50g

Finding a value given 1:n (or n:1)

Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

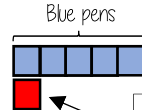
Model the Question

Blue : Red

5 : 1

□ = one part = 10 pens

Blue pens



Red pens

One unit = 10 pens

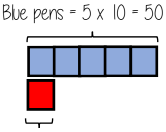
Put back into the question

Blue : Red

× 10 5 : 1 × 10

50: 10

Blue pens = 5 x 10 = 50 pens



Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

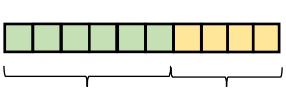
Simplifying a ratio

Cancel down the ratio to its lowest form

Find the biggest common factor that goes into all parts of the ratio
For 6 and 4 the biggest factor (number that multiplies into them is 2)

"For every 6 days of rain there are 4 days of sun"

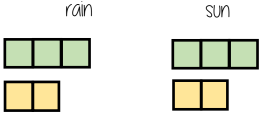
6:4



rain sun

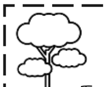
÷ by 2

3:2



"For every 3 days of rain there are 2 days of sun

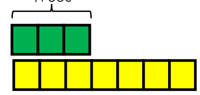
Ratio as a fraction



Trees: Flowers

3 : 7

Trees



Flowers

Ratio

There are 3 parts for trees

Fraction of trees

$$\frac{\text{Number of parts of in group}}{\text{Total number of parts}} \qquad \frac{3}{10}$$

Fraction

Tree parts 3 + Flower parts 7 = 10

Ratio 1:n (or n:1)

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that this part has to be 1 unit Therefore Divide by 4

4 : 20

1 : 5

This side has to be divided by 4 too – to keep in proportion

H the n part does not have to be an integer for this type of question

Units are important:

When using a ratio – all parts should be in the same units

Useful Conversions

$$mm \xrightleftharpoons[\div 10]{\times 10} cm \xrightleftharpoons[\div 100]{\times 100} m \xrightleftharpoons[\div 1000]{\times 1000} km$$

$$g \xrightleftharpoons[\div 1000]{\times 1000} kg$$

$$ml \xrightleftharpoons[\div 1000]{\times 1000} L$$

RATIO & PROPORTION... Proportion & scale

What do I need to be able to do?

- Direct proportion
- Conversion graphs
- Convert between currencies
- Direct proportion graphs
- Similar shapes
- Convert metric units
- Scale diagrams
- Interpret maps using scale and ratios

Keywords

Tier 2:

Proportion: a statement that links two ratios
Variable: a part that the value can be changed
Axes: horizontal and vertical lines that a graph is plotted around
Approximation: an estimate for a value
Scale Factor: the multiple that increases/ decreases a shape in size
Currency: the system of money used in a particular country
Conversion: the process of changing one variable to another
Scale: the comparison of something drawn to its actual size.

Direct Proportion

As one variable changes the other changes at the same rate.

4 cans of pop = £2.40

This is a multiplicative change

4 cans of pop = £2.40 $\xrightarrow{\times 3}$ 12 cans of pop = £7.20

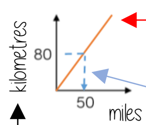
4 cans of pop = £2.40 $\xrightarrow{\times 0.5}$ 2 cans of pop = £1.20

This multiplier is the same In the same way that this would be for ratio

Sometimes this is easiest if you work out how much one unit is worth first
e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

Conversion between currencies

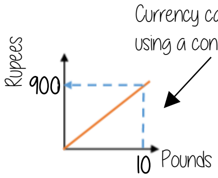
£1 = 90 Rupees

Currency is directly proportional

For every £1 I have 90 Rupees

£1 = 90 Rupees $\xrightarrow{\times 10}$ £10 = 900 Rupees

Currency can be converted using a conversion graph

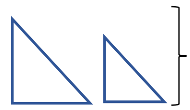


Convert 630 Rupees into Pounds

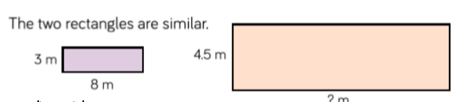
£1 = 90 Rupees $\xrightarrow{\times 7}$ £7 = 630 Rupees

630 ÷ 90 = 7

Ratio between similar shapes



Angles in similar shapes do not change.
e.g if a triangle gets bigger the angles can not go above 180°



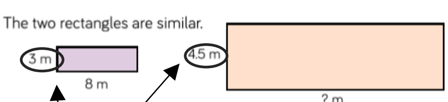
Corresponding sides

3m : 4.5m $\xrightarrow{\div 3}$ 1 m : 1.5 m

8m : 12m $\xrightarrow{\div 8}$ 1 m : 1.5 m

Note
Simplify to the same ratio

Understand Scale Factor



3 x 1.5 = 4.5

This is a multiplicative change

Use **corresponding** sides to calculate a scale factor

Missing length
8 x 1.5 = 12m

Scale factor can also be calculated by:

$$\frac{\text{Bigger corresponding side}}{\text{Smaller corresponding side}}$$

Small corresponding side $\xrightarrow{\times \text{SF}}$ Big corresponding side

Big corresponding side $\xrightarrow{\div \text{SF}}$ Small corresponding side

Draw and interpret scale diagrams

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

Image	Real life
1cm	30cm
10cm	300cm

(× 10)

The car in real life is 210cm

Image	Real life
1cm	30cm
7cm	210 cm

(× 7)

Interpret maps with scale factors

mm $\xrightarrow{\div 10}$ cm $\xrightarrow{\div 100}$ m $\xrightarrow{\div 1000}$ km

km $\xrightarrow{\times 1000}$ m $\xrightarrow{\times 100}$ cm $\xrightarrow{\times 10}$ mm

1 cm : 250 m

Ratios need to be in the same units

1 cm : 250m

250 x 100 = 25000

1 cm : 25000cm

For every 1cm on my map is 25000cm in real life.

NUMBER… Algebraic manipulation

What do I need to be able to do?

- Form algebraic expressions
- Identify and use formulae, expressions, identities and equations
- Simplify expressions
- Use directed number with algebra
- Substitution with directed number
- Expand a single bracket
- Factorise into a single bracket
- Expand single brackets and simplify
- Expand double brackets (E)
- Factorise quadratic expressions (E)

Keywords

Tier 2:

Variable: a symbol for a number we don't know yet

Expression: numbers, symbols and operators grouped together to show the value of something

Identity: An equation where both sides have variables that cause the same answer includes ≡

Substitute: replace one variable with a number or new variable

Equivalent: something of equal value

Coefficient: a number used to multiply a variable

Equation: a statement that the values of two mathematical expressions are equal (indicated by the sign =)

Formula: A mathematical relationship/ rule given in symbols

Tier 3:

Highest Common Factor (HCF): the biggest factor (or number that multiplies to give a term)

Using letters to represent numbers

5 + 5 + 5 3 x 5 5 x 3	y + y + y + y y x 4 4 x y 4y	20 ÷ h $\frac{20}{h}$
Addition and multiplication can be done in any order **Commutative calculations**	4 lots of 'y'	20 shared into 'h' number of groups

Substitution into expressions

4y ← 4 lots of 'y'

If y = 7 this means the expression is asking for 4 'lots of' 7

4 x 7 OR 7 + 7 + 7 + 7 OR 7 x 4 $\boxed{= 28}$

eg: y – 2
= 7 – 2 = 5

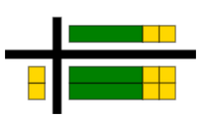
$\boxed{2(x + 3)}$ Put the expression into a function machine

INPUT → + 3 → x 2 → OUTPUT

If x = 10   10 + 3 = 13…. 13 x 2 = 26

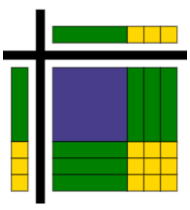
Expand double brackets

$2\,(x + 2) \equiv 2x + 4$



Algebra tiles can represent a binomial expansion
Has two terms

$(x + 3)(x + 3) \equiv x^2 + 6x + 9$



This is a quadratic.
It has four terms which simplified to three terms

The order of the brackets has no impact on the outcome.
eg $(x + 3)(3 + x)$

Collecting like terms ≡ symbol

The ≡ symbol means equivalent to.
It is used to identify equivalent expressions

Collecting like terms

Only **like terms** can be combined

4x +5b –2x +10b

$\boxed{4x}$ $\boxed{+5b}$ $\boxed{-2x}$ $\boxed{+10b}$

2x + 15b

Common misconceptions

$2x + 3x^2 + 4x \;\equiv\; 6x + 3x^2$

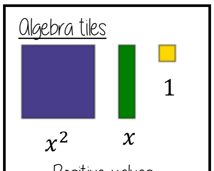
Although they both have the x variable x2 and x terms are un-like terms so can not be collected

Factorising Quadratics

When a quadratic expression is in the form $x^2 + bx + c$ find the two numbers that **add to give b** and **multiply to give c**.

$x^2+7x+10 = (x+5)(x+2)$

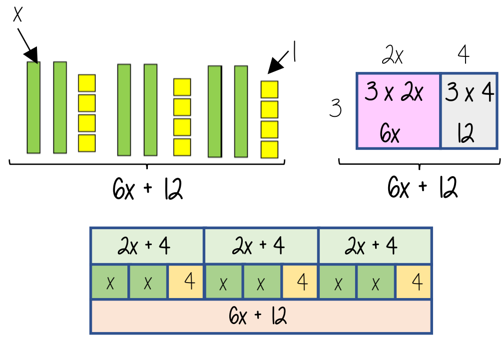
(because 5 and 2 add to give 7 and multiply to give 10)



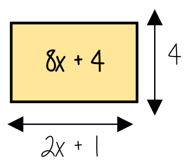
Expand single brackets

3 (2x + 4)

Different representations of 3(2x+4) = 6x + 12



Factorise into a single bracket



← Try and make this the **highest common factor**

$\boxed{\text{The two values multiply together (also the area) of the rectangle}}$

8x + 4 ≡ 4 (2x + 1)

Note:
8x + 4 ≡ 2(4x + 2)
This is factorised but the HCF has not been used

Directed numbers

+ +	→	+
– –	→	+
+ –	→	–
– +	→	–

eg a = -5 and b = 2

a^2 = a x a = -5 x -5 = 25

b + a = 2 + - 5 = -3

Brackets around negative substitutions helps remove calculation errors

2a – b = 2 x 5 – (-4) = 10 + 4 = <u>14</u>

ALGEBRA... Coordinates & graphs

What do I need to be able to do?

- Coordinates in all four quadrants
- Lines parallel to the axes
- Table of values
- Recognise and use the line y=x
- Lines of the form y=mx
- Link y=mx to direct proportion (E)
- Introduce gradient (y=mx)
- Lines with a negative gradient
- Lines of the form y=x+c
- Lines of the form y=mx+c
- Find the midpoint of a line segment (E)
- Solve problems with coordinates and graphs (E)
- Quadratic graphs (E)

Keywords

Tier 2:

Quadrant: four quarters of the coordinate plane.

Coordinate: a set of values that show an exact position.

Horizontal: a straight line from left to right (parallel to the x axis)

Vertical: a straight line from top to bottom (parallel to the y axis)

Origin: (0,0) on a graph. The point the two axes cross

Parallel: Lines that never meet

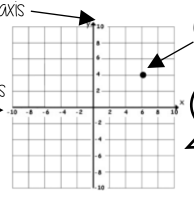
Gradient: The steepness of a line

Intercept: Where lines cross

Coordinates in four quadrants

y-axis

x-axis



Coordinate (x, y) (6, 4)

From the origin this coordinate is 6 places along the positive x axis and 4 places up the positive y axis.

(x, y)

Always the position on the x axis first

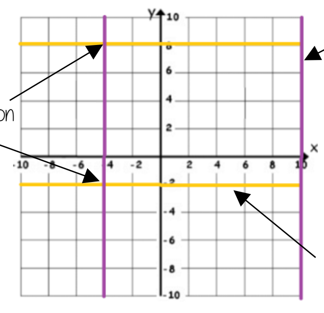
Always the position on the y axis second

(0, a) Will be always be a point on the y axis (a can be any number)

(a, 0) Will be always be a point on the x axis (a can be any number)

Lines parallel to the axes

Intersection points



All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

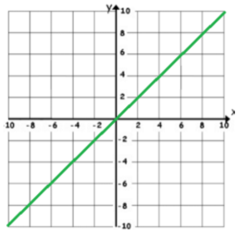
Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

e.g (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

Recognise and use the line y=x



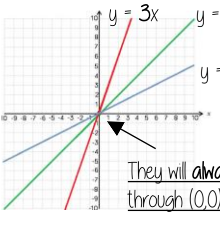
This means the x and the y coordinate have the same value

Examples of coordinates on this line: (0, 0) (-3, -3) (8, 8)

The axes **scale is important** – if the scale is the same y = x will be a straight line at 45°

Recognise and use the lines y=mx

The value of k changes the steepness of the line



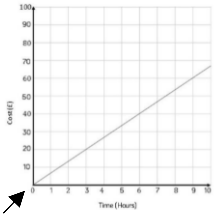
Note: y =x is the same as y= 1x

The bigger the value of k the **steeper** the line will be.

They will **always** go through (0,0)

The closer to 0 the value of k the closer the line will be to the x axis.

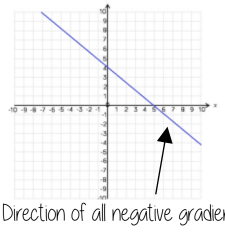
Direct Proportion using y=kx



The line must be straight to be directly proportional – variables increase at the same rate **k**

Direct proportion graphs always start at (0,0) as they are describing relationships between two variables

Lines with negative gradients

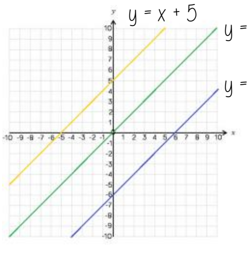


Any straight-line graph with a negative x value has a negative gradient.

Eg y = -2x
y = -x   y + x = 12

Direction of all negative gradients

Lines in the form y = x + a



All the lines are **parallel** because the gradients are the same

y = x + a

This is the line y=x when the y and x coordinate are the same

This shows the translation of that line.
e.g y = x + 5
Is the line y=x moved 5 places up the graph

5 has been added to each of the x coordinates

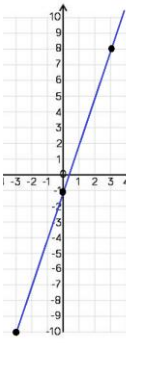
Plotting y = mx + c graphs

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

NUMBER... Multiply and Divide Fractions

What do I need to be able to do?

- Multiply a fraction by an integer
- Multiply fractions
- Understand reciprocals
- Divide a fraction by an integer
- Divide an integer by a fraction
- Divide a fraction by a unit fraction
- Divide fractions
- Multiply and divide mixed numbers
- Multiply and divide algebraic fractions (E)

Keywords

Tier 2:

Whole: a positive number including zero without any decimal or fractional parts.

Dividend : the amount you want to divide up.

Divisor: the number that divides another number.

Quotient: the answer after we divide one number by another. e.g. dividend÷ divisor = quotient

Reciprocal: a pair of numbers that multiply together to give 1

Tier 3:

Numerator : the number above the line on a fraction. The top number. Represents how many parts are taken

Denominator: the number below the line on a fraction. The number represent the total number of parts.

Unit Fraction: a fraction where the numerator is one and denominator a positive integer.

Non-unit Fraction: a fraction where the numerator is larger than one.

Commutative: an operation is commutative if changing the order does not change the result.

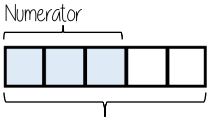
Representing a fraction

$\frac{\text{Numerator}}{\text{Denominator}}$

$\frac{3}{5}$

Number of parts represented

Numerator



Number of parts to make up the whole

Denominator

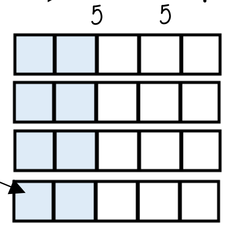
ALL PARTS of a fraction are of equal size

Repeated addition = multiplication by an integer

$4 \times \frac{2}{5} \rightarrow \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$

Integer (Whole number)

Each part represents $\frac{1}{5}$



How many parts are shaded?

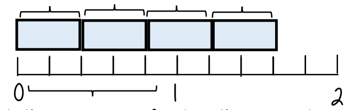
What each part represents

Revisit

When adding fractions with the same denominator = add the numerators

$= \frac{8}{5} \leftarrow \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$

$= 1\frac{3}{5}$



Each whole is split into the same number of parts as the denominator

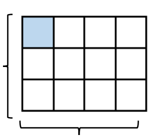
Multiplying unit fractions

$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

Parts shaded

Total number of parts in the diagram

Modelled:



Multiplying non-unit fractions

Shade in 3 parts

Repeat it on this many rows

$\frac{3}{4} \times \frac{2}{3}$

This many columns

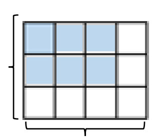
This many rows

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$

Parts shaded

Total number of parts in the diagram

Modelled:



Quick Multiplying and Cancelling down

$\frac{\cancel{3}^{1}}{5} \times \frac{4}{\cancel{9}_{3}}$

The 3 and the 9 have a common factor and can be simplified

Quick Solving

Multiply the numerators

Multiply the denominators

$\frac{1 \times 4}{5 \times 3} = \frac{4}{15}$

The reciprocal

When you multiply a number by its reciprocal the answer is always 1

$3 \times \frac{1}{3} = 1$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

The reciprocal of 3 is $\frac{1}{3}$ and vice versa

Reciprocals for division

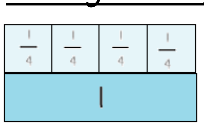
e.g.

$5 \div \frac{1}{4} = 20$

$5 \times 4 = 20$

Multiplying by a reciprocal gives the same outcome

Dividing an integer by a unit fraction



$1 \div \frac{1}{4} = 4$

How many quarters are in 1?

"There are 4 quarters in 1 whole. Therefore, there are 20 quarters in 5 wholes"

$5 \div \frac{1}{4} = 20$

Dividing any fractions

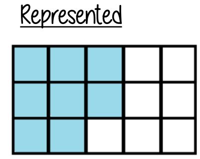
Remember to use reciprocals

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

Represented



$= \frac{8}{15}$

GEOMETRY… Symmetry and reflection

What do I need to be able to do?

- Line symmetry
- Rotational symmetry
- Reflect a shape in a horizontal or vertical line
- Reflect a shape in a diagonal line
- Reflect a shape given equation of a line (E)
- Describe a reflection (E)

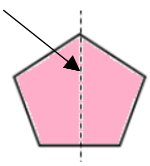
Keywords

Tier 2:

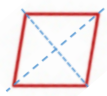
Mirror line: a line that passes through the centre of a shape with a mirror image on either side of the line
Line of symmetry: same definition as the mirror line
Reflect: mapping of one object from one position to another of equal distance from a given line.
Rotate: a rotation is a circular movement.
Vertex: a point where two or more-line segments meet.
Perpendicular: lines that cross at 90°
Horizontal: a straight line from left to right (parallel to the x axis)
Vertical: a straight line from top to bottom (parallel to the y axis)

Lines of symmetry

Mirror line (line of reflection)



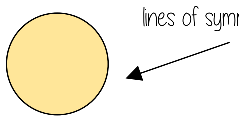
Rhombus
two lines of symmetry



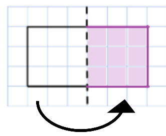
Parallelogram
No lines of symmetry

Shapes can have more than one line of symmetry… This regular polygon (a regular pentagon has 5 lines of symmetry)

A circle has an infinite amount of lines of symmetry

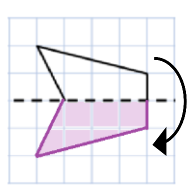


Reflect horizontally/ vertically (1)



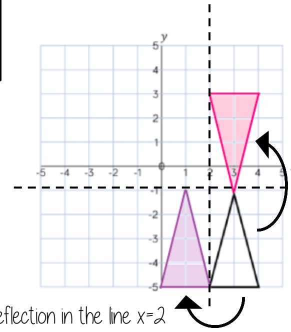
Note: a reflection doubles the area of the original shape

Reflection in a vertical line



Reflection in a horizontal line

Reflection on an axis grid

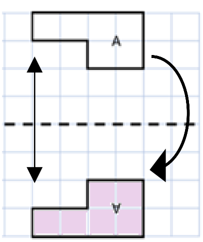


Reflection in the line y=-2

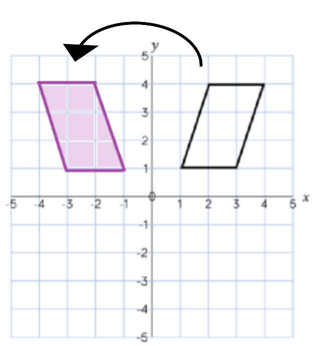
Reflection in the line x=2

Reflect horizontally/ vertically (2)

All points need to be the same distance away from the line of reflection



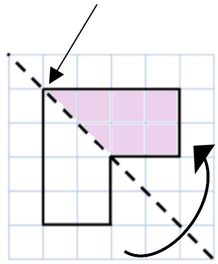
Reflection in the line y axis – this is also a reflection in the line x=0



Lines parallel to the x and y axis
REMEMBER
Lines parallel to the x-axis are y = ____
Lines parallel to the y-axis are x = ____

Reflect Diagonally (1)

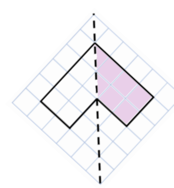
Points on the mirror line don't change position

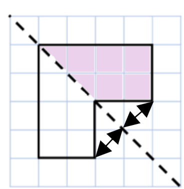


Fold along the line of symmetry to check the direction of the reflection

Turn your image

If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)



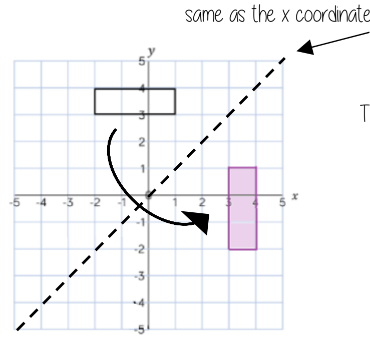


Drawing perpendicular lines

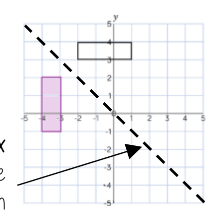
Perpendicular lines to and from the mirror line can help you to plot diagonal reflections

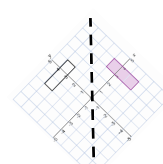
Reflect Diagonally (2)

This is the line **y = x** (every y coordinate is the same as the x coordinate along this line)



This is the line **y = - x**
The x and y coordinate have the same value but opposite sign





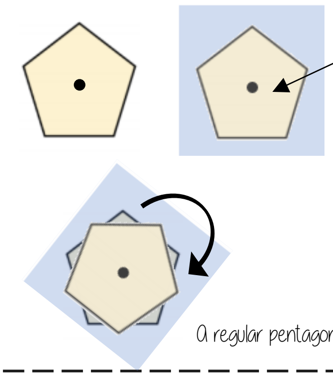
Turn your image

If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)

GEOMETRY… Symmetry and reflection

Rotational Symmetry

Tracing paper helps check rotational symmetry



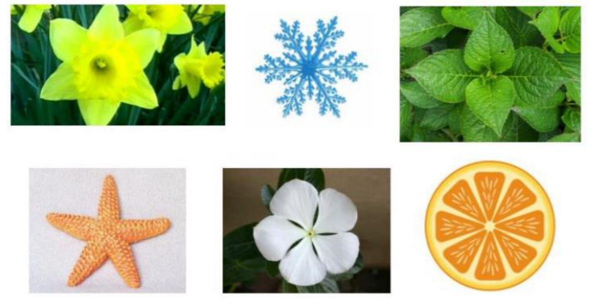
1 Trace your shape (mark the centre point)

2 Rotate your tracing paper on top of the original through 360°

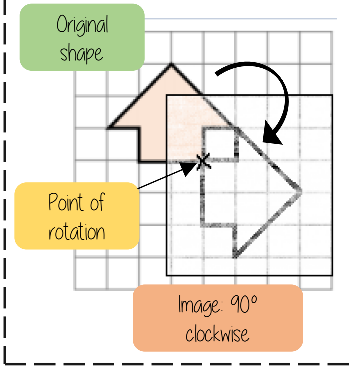
3 Count the times it fits back into itself

a regular pentagon has rotational symmetry of order 5

Rotational symmetry in nature.



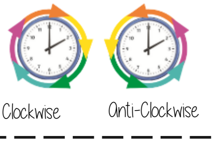
Rotate from a point (in a shape)



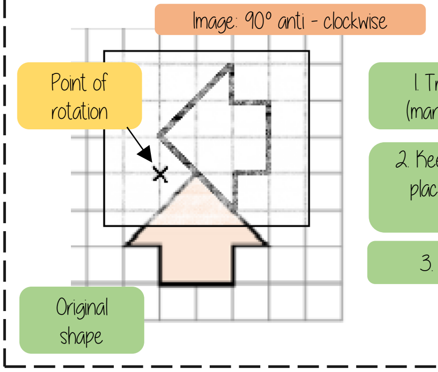
1 Trace the original shape (mark the point of rotation)

2 Keep the point in the same place and turn the tracing paper

3 Draw the new shape



Rotate from a point (outside a shape)

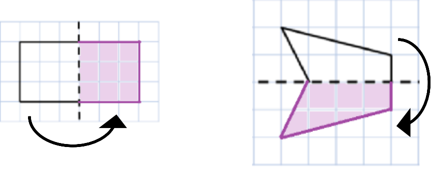


1 Trace the original shape (mark the point of rotation)

2 Keep the point in the same place and turn the tracing paper

3 Draw the new shape

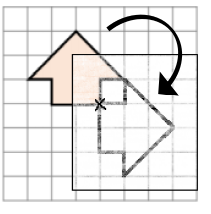
Compare rotations and reflections



Reflections are a mirror image of the original shape.

Information needed to perform a reflection:

- Line of reflection (Mirror line)



Rotations are the movement of a shape in a circular motion

Information needed to perform a rotation:

- Point of rotation
- Direction of rotation
- Degrees of rotation

GEOMETRY... Area, volume and density

What do I need to be able to do?

- Name 2-D and 3-D shapes
- Area of a 2-D shape
- Area of a compound shape
- Recognise prisms
- Volume of cubes and cuboids
- Convert metric units of mass and capacity
- Understand the units of density, mass and volume
- Problems with density, mass and volume

Keywords

Tier 2:

2D: two dimensions to the shape e.g. length and width

3D: three dimensions to the shape e.g. length, width and height

Edge: a line on the boundary joining two vertex

Face: a flat surface on a solid object

Density: a measure of how compact a substance is. The more material is in that volume, the denser it is.

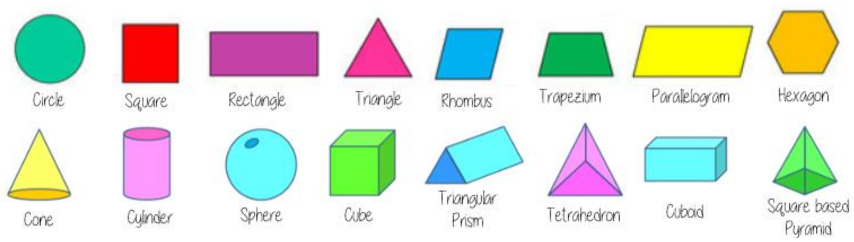
Cross-section: a view inside a solid shape made by cutting through it

Prism: a solid object with two identical ends and flat sides.

Tier 3:

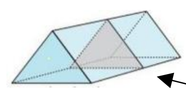
Vertex: a point where two or more line segments meet

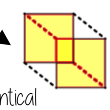
Name 2D & 3D shapes



Recognise prisms

A solid object with two identical ends and flat sides



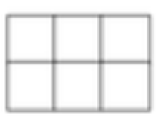


The cross section will also be identical to the end faces.

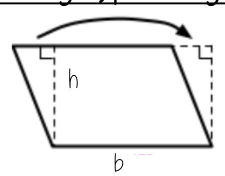
A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

Area – rectangles, triangles, parallelograms

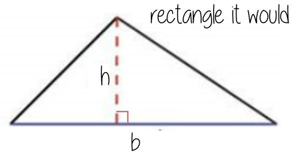
A triangle is half the size of the rectangle it would fit in



Rectangle
Base x Height



Parallelogram/ Rhombus
Base x Perpendicular height

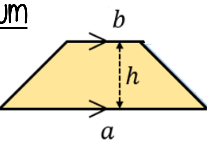


Triangle
½ x Base x Perpendicular height

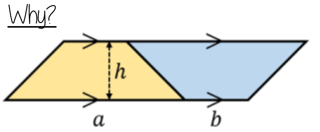
Area of a trapezium

Area of a trapezium

$$\frac{(a+b)\times h}{2}$$



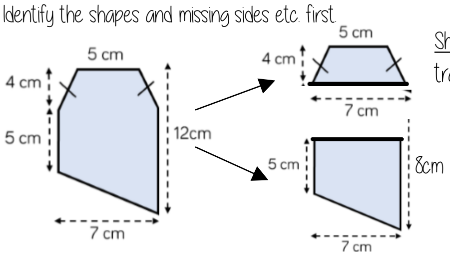
Why?



Two congruent trapeziums make a parallelogram
New length (a + b) x height
Divide by 2 to find area of one

Compound shapes

To find the area compound shapes often need splitting into more manageable shapes first. Identify the shapes and missing sides etc. first.



Shape A - Isosceles trapezium

Shape B- nonstandard trapezium

Shape A + Shape B = total area

$$\frac{(5+7)\times 4}{2} + \frac{(5+8)\times 7}{2} = 24 + 45.5 = 69.5cm^2$$

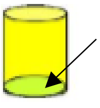
Units

Volumes

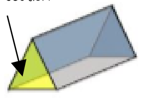
Volume is the 3D space it takes up – also known as capacity if using liquids to fill the space.

Counting cubes

Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape.



Cross section



Cubes/ Cuboids = base x width x height

Remember multiplication is commutative
Height can also be described as depth

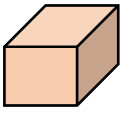
Prisms and cylinders
= area cross section x height

Density, Mass, Volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

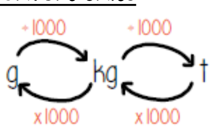
$$\text{volume} = \frac{\text{mass}}{\text{density}}$$

$$\text{mass} = \text{volume} \times \text{density}$$



volume of prism = Area of cross section × Depth

Convert units



REMEMBER!
1000g = 1kg
1000kg = 1 tonne(t)

Example 1

Convert 1.458t to g

$\xrightarrow{\times 1000}$

1.458t = 1458kg

$\xrightarrow{\times 1000}$

1458kg = 1458000g

Example 2

Convert 15600 to kg

$\xrightarrow{\div 1000}$

15600g = 15.6kg

ALGEBRA... Equations & Inequalities

What do I need to be able to do?

- Solve 1- and 2-step equations
- Solve more complex equations
- Solve fractional equations
- Form and solve equations
- Solve equations with unknowns on both sides
- Understand and use inequalities
- Inequalities on a number line
- Solve inequalities
- Form and solve inequalities
- Solve inequalities with unknowns on both sides (E)

Keywords

Tier 2:

Simplify: grouping and combining similar terms

Substitute: replace a variable with a numerical value

Equivalent: something of equal value

Coefficient: a number used to multiply a variable

Product: multiply terms

Inequality: an inequality compares who values showing if one is greater than, less than or equal to another

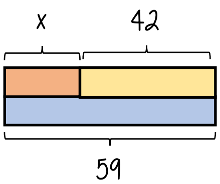
Solve one step equations (+/-)

There is more to this than just spotting the answer

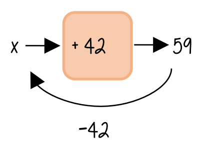
$x + 42 = 59$

$x + 42 = 59$
$42 + x = 59$

$59 - x = 42$
$59 - 42 = x$



Don't forget you know how to use function machines

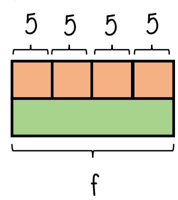


Solve one step equations (x/÷)

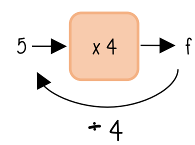
$\frac{f}{4} = 5$

$f \div 4 = 5$
$f \div 5 = 4$

$5 \times 4 = f$
$4 \times 5 = f$

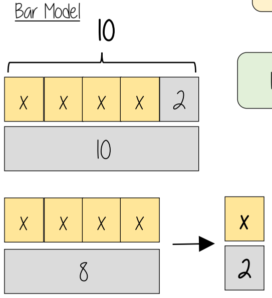


Don't forget you know how to use function machines



Two-step equations

Bar Model

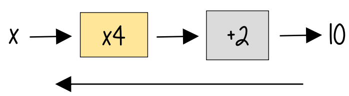


$4x + 2 = 10$

$10 - 4x = 2$

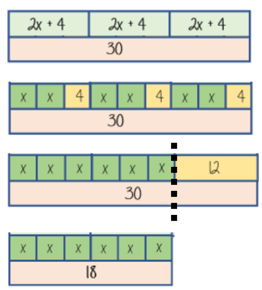
Representing the same question (use fact families)

Function machine



Inverse operations to find x

Solve equations with brackets



$3(2x + 4) = 30$

$6x + 12 = 30$

$-12 \quad -12$

$6x = 18$

$\div 6 \quad \div 6$

$3(2x + 4) = 30$

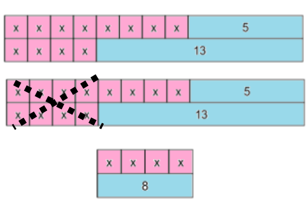
Expand the brackets

Substitute to check your answer. This could be negative or a fraction or decimal

$x = 3$

Equations: unknown on both sides

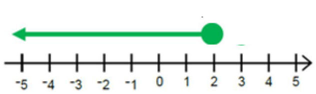
$8x + 5 = 4x + 13$



$$8x + 5 = 4x + 13$$
$$-4x \qquad -4x$$
$$4x + 5 = 13$$
$$-5 \qquad -5$$
$$4x = 8$$
$$\div 4 \qquad \div 4$$
$$x = 2$$

Inequalities: unknown on both sides

$8x + 5 \leq 4x + 13 \rightarrow x \leq 2$



Ony value 2 or less will satisfy this inequality

Form and solve inequalities

Two more than treble my number is greater than 11

Find the possible range of values

Form

x → x3 → +2 → 11

$3x + 2 > 11$

Solve

x ← ÷3 ← -2 ← 11

$x > 3$

Check

This would suggest any value bigger than 3 satisfies the statement

$3 \times 3 + 2 = 11$ ✓ $\quad 10 \times 3 + 2 = 32$ ✓

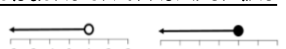
Simple Inequalities

$<$ less than
$>$ More than
$\leq$ Less than or equal to
$\geq$ More than or equal to

$x < 10$
Say this out loud
"x is a value less than 10"

$10 > x$
Say this out loud
"10 is more than the value"

Solutions on a number line



$x < 1 \qquad x \leq 1$

Both represent values less than 1

Includes the value 1

$x > 1 \qquad x \geq 1$

Both represent values more than 1

Includes the value 1

● Includes the value it sits above

○ Does NOT include the value it sits above

$-1 < x \leq 3$

This includes the integer values 0,1,2,3

NUMBER…

Percentages

What do I need to be able to do?

- Percentage of an amount
- Convert between percentages and decimals
- Use multipliers to find percentages
- Convert between decimals and percentages greater than 1
- Percentage increase using a multiplier
- Percentage decrease using a multiplier
- Percentage increase and decrease using a multiplier
- Express one number as a fraction or a percentage of another (calculator)
- Express one number as a fraction or a percentage of another (non-calculator)
- Percentage change
- Find the original value given a percentage
- Choose appropriate methods to solve percentage problems

Keywords

Tier 2:

Percent: parts per 100 – written using the % symbol

Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.

Fraction: a fraction represents how many parts of a whole value you have.

Equivalent: of equal value.

Reduce: to make smaller in value.

Growth: to increase/ to grow.

Integer: whole number, can be positive, negative or zero.

Invest: use money with the goal of it increasing in value over time (usually in a bank).

Convert FDP

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares → 70 "hundredths" = 7 "tenths" 0.7 → 70 hundredths = 70%

Using a calculator

■/□ → S⇆D → Convert to a decimal → × 100 converts to a percentage

This will give you the answer in the simplest form

Be careful of recurring decimals

eg $\frac{1}{3}$ = 0.3333333

= $0.\dot{3}$

The dot above the 3

Percentage change

I bought a phone for £200. A year later sold it for £125.

100%

£200

£125

Percentage loss

All values of change compare to the ORIGINAL value

$\frac{75}{200} \times 100 = 37.5\%$

$\frac{\text{Difference in value}}{\text{Original value}} \times 100$

I bought a house for £180,000, I later sold it for £216,000.

100%

£180,000

Percentage profit

Money made (profit value)

$\frac{36000}{180000} \times 100 = 20\%$

Percentage decrease: Multipliers

100%

42%

Decrease by 58%

100% − 58% = 42%

1.00 – 0.58 = 0.42

Multiplier Less than 1

Percentage increase: Multipliers

100%

12%

Increase by 12%

100% + 12% = 112%

1.00 + 0.12 = 1.12

Multiplier More than 1

Percentage of amount

Find 60% of £60

£60

£12 £12 £12 £12 £12

£36

10% of £60 = £6

50% of £60 = £30

60% of £60 = £36

60% of £60

= 0.6 x 60

= £36

Express as a % - Non-calculator

Percent – per hundred

7 per every 10 are orange $\frac{7}{10}$

This means that 70 per every 100 are orange $\frac{70}{100}$

70%

27 per every 50 shaded $\frac{27}{50}$

54 per every 100 shaded $\frac{54}{100}$

54%

Denominator 100

Equivalent fractions

Express as a % - Calculator

Rosie

$\frac{13}{30}$ → ■/□ → $\frac{13}{30}$ → × 100 → 43.3333…%

43%

This the same as 13 ÷ 30

Can't use equivalence easily to find 'per hundred'

Decimal percentages are still a percentage

Find the original value

Percentage calculations

Original amount × Multiplier = Final Value

In a test Lucy scored 60% of her questions correctly. Her score was 24. How many questions were on the test.

Original x 0.6 = 24

60%

24

24 ÷ 0.6 = 40 marks

10% = 6

100% = 40

Total questions on test

A car sold for a profit £3000 with a profit of 20%. How much was the car originally?

100%

20%

£3000

Original x 1.2 = 3000

120% = £3000

10% = £250

100% = £2500

Choose appropriate method

The language and wording of the question is the key

Have you represented the question in a bar model?

Can you use a calculator?

ALGEBRA...

Indices

What do I need to be able to do?

- Add and subtract expressions with indices
- Multiply and divide expressions with indices
- Addition law for indices
- Subtraction law for indices
- Addition and subtraction laws for indices
- Powers of powers (E)
- Negative indices (E)
- Fractional indices (E)

Keywords

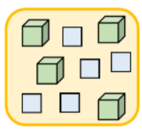
Tier 2:

Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Coefficient: The number used to multiply a variable
Simplify: To reduce a power to its lowest term
Product: Multiply

Addition/ Subtraction with indices

Coefficient → $5x^2 + 4x^4$ ← Power

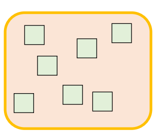
$\underbrace{\underbrace{5x^2}_{\text{Term}} + \underbrace{4x^4}_{\text{Term}}}_{\text{Expression}}$ →



Each square represents x^2 and each cube represents x^4

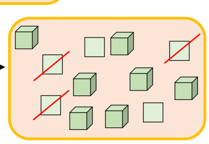
Only similar terms can be simplified
If they have different powers, they are unlike terms

$5x^2 + 2x^2$ →



→ $7x^2$

$5x^2 + 6x^4 - 3x^2 + x^4$ →



→ $2x^2 + 7x^4$

Multiply expressions with indices

$4b \times 3a$
$\equiv 4 \times b \times 3 \times a$
$\equiv 4 \times 3 \times b \times a$
$\equiv 12ab$

$5t \times 9t$
$\equiv 5 \times t \times 9 \times t$
$\equiv 5 \times 9 \times t \times t$
$\equiv 45t^2$

$2b^4 \times 3b^2$
$\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b$
$\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b$
$\equiv 6b^6$

There are often misconceptions with this calculation but break down the powers

Addition/ Subtraction laws for indices

$3^5 \times 3^2$
$= \underbrace{(3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)}$ → 3^7

The base number is all the same so the terms can be simplified

Addition law for indices

$$a^m \times a^n = a^{m+n}$$

$3^5 \div 3^2$ → 3^3

$$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \rightarrow \frac{3^3}{3^0} \rightarrow \frac{3^3}{1}$$

Subtraction law for indices

$$a^m \div a^n = a^{m-n}$$

Divide expressions with indices

$$\frac{24}{36} \rightarrow \frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 2 \times \cancel{3}} \rightarrow \frac{2}{3}$$

$$\frac{5a^3b^2}{15ab^6} \rightarrow \underbrace{\frac{\cancel{5} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b}}{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times b \times b \times b \times b}} \rightarrow \frac{a^2}{3b^4}$$

Cross cancelling factors shows cancels the expression

$\frac{23a^7y^2}{5db^6}$ This expression cannot be divided (cancelled down) because there are no common factors or similar terms

NUMBER...

Standard Form

What do I need to be able to do?

- Positive and negative powers of 10
- Numbers greater than 1 in standard form
- Numbers between 0 and 1 in standard form
- Standard form on a calculator

Keywords

Tier 2:
Standard (index) Form: A system of writing very big or very small numbers
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Negative: A value below zero.
Tier 3:
Commutative: an operation is commutative if changing the order does not change the result.

Positive powers of 10

1 billion – 1 000 000 000
$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

Addition rule for indices $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices $10^a \div 10^b = 10^{a-b}$

Numbers between 0 and 1

0.054
$= 5.4 \times 10^{-2}$

1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^0	10^{-1}	10^{-2}	10^{-3}
0	0	5	4

A negative power does not mean a negative answer – it means a number closer to 0

Standard form with numbers > 1

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Example
3.2×10^4
$= 3.2 \times 10 \times 10 \times 10 \times 10$
$= 32000$

Non-example
0.8×10^4
$5.3 \times 10^{0.7}$

Negative powers of 10

0.001
$1 \times \frac{1}{1000}$
1×10^{-3}

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
0	0	0	0	1

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Order numbers in standard form

10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}

6.4×10^{-2}	2.4×10^2	3.3×10^0	1.3×10^{-1}
0.064	240	1	0.13

Look at the power first
will the number be = > or < than 1

Use a place value grid to compare the numbers for ordering

Mental calculations

$6.4 \times 10^2 \times 1000$ Not in Standard Form
$= 6.4 \times 10^2 \times 10^3$ Use addition for indices rule
$= 6.4 \times 10^5$

$8 \times 10^5 \times 3$
$= 24 \times 10^5$ Not in Standard Form
$= 2.4 \times 10^1 \times 10^5$ Use addition for indices rule
$= 2.4 \times 10^6$

$(2 \times 10^3) \div 4$ Divide the values
$= (2 \div 4) \times 10^3$
$= 0.5 \times 10^3$

Remember the layout for standard form

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$$6 \times 10^5 + 8 \times 10^5$$

Method 1
$= 600000 + 800000$
$= 1400000$
$= 1.4 \times 10^5$

More robust method
Less room for misconceptions
Easier to do calculations with negative indices
Can use for different powers

Method 2
$= (6 + 8) \times 10^5$
$= 14 \times 10^5$ ← This is not the final answer
$= 1.4 \times 10^1 \times 10^5$
$= 1.4 \times 10^5$

Only works if the powers are the same

Multiplication and division

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$ ← Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \times 10^5 \div 10^3$

$= 5 \times 10^2$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

Revisit addition and subtraction laws for indices – they are needed for the calculations

Addition law for indices
$a^m \times a^n = a^{m+n}$

Subtraction law for indices
$a^m \div a^n = a^{m-n}$

Using a calculator

$14 \times 10^5 \times 3.9 \times 10^3$

Use a calculator to work out this question to a suitable degree of accuracy

Input 14 and press **x10^x** Then press 5 (for the power)
Press **×**
Input 3.9 and press **x10^x** Then press 3 (for the power)
Press **=**

This gives you the solution

Click calculator for video tutorial

To put into standard form and a suitable degree of accuracy
Press **SHIFT** **SETUP** and then press 7 for sci mode
Choose a degree of accuracy so in most cases press 2

Answer: 5.5×10^8

STATISTICS… Interpret & represent data

What do I need to be able to do?

- Types of data
- Outliers and errors
- Averages and range
- Choose the most appropriate average
- Compare distributions using average and the range
- Averages from an ungrouped frequency table
- Represent and interpret grouped discrete data
- Represent and interpret continuous data grouped into equal classes
- Mean and mode from a grouped frequency table (E)

Keywords

Tier 2:

Spread: the distance/ how spread out/ variation of data
Average: a measure of central tendency – or the typical value of all the data together
Frequency: the number of times the data values occur
Outlier: a value that stands apart from the data set
Quantitative: numerical data
Qualitative: descriptive information, colours, genders, names, emotions etc.
Continuous: quantitative data that has an infinite number of possible values within its range.
Discrete: quantitative or qualitative data that only takes certain values

Mean, Median, Mode

The Mean

A measure of average to find the central tendency… a typical value that represents the data

24, 8, 4, 11, 8,

Find the sum of the data (add the values) 55

Divide the overall total by how many pieces of data you have 55 ÷ 5

Mean = 11

The Median

The value in the centre (in the middle) of the data

24, 8, 4, 11, 8,

Put the data in order 4, 8, 8, 11, 24

Find the value in the middle 4, 8, 8, 11, 24

Median = 8

NOTE: If there is no single middle value find the mean of the two numbers left

The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8,

This can still be easier if it the data is ordered first

4, 8, 8, 11, 24

Mode = 8

Identify outliers

Outliers are values that stand well apart from the rest of the data

Outliers can have a big impact on range and mean.
They have less impact on the median and the mode

Height in cm											
152	150	142	158	182	151	153	149	156	160	151	144

Where an outlier is identified try to give it some context. This is likely to be a taller member of the group. Could this be an older student or a teacher?

Sometimes it is best to not use an outlier in calculations

Choosing the appropriate average

Which average best represents the weekly wage?

Here are the weekly wages of a small firm

£240	£240	£240	£240	£240
£260	£260	£300	£350	£700

The Mean = £307
The Median = £250
The Mode = £240

Put the data back into context

Mean/Median – too high (most of this company earn £240)
Mode is the best average that represents this wage

It is likely that the salaries above £240 are more senior staff members – their salary doesn't represent the average weekly wage of the majority of employers

Averages from a table

Non-grouped data

Number of Siblings	0	1	2
Frequency	6	8	6
Subtotal	0	8	12

Overall Frequency: 20

Total number of siblings: 20

The data in a list: 0,0,0,0,0,0,1,1,1,1,1,1,1,1,2,2,2,2,2,2

Mean: $\frac{\text{total number of siblings}}{\text{Total frequency}} = 1$

Grouped data

x Weight(g)	Frequency	Mid Point	MP x Freq
$40 < x \leq 50$	1	45	45
$50 < x \leq 60$	3	65	195
$60 < x \leq 70$	5	65	325

Overall Frequency: 9

Overall Total : 565

Mean: 62.8g

The data in a list: 45, 55, 55, 55, 65, 65, 65, 65, 65

Comparing distributions

Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency.

Here are the number of runs scored last month by Lucy and James in cricket matches

Lucy: 45, 32, 37, 41, 48, 35
James: 60, 90, 41, 23, 14, 23

Lucy
Mean: 39.6 (1dp), **Median**: 38 **Mode**: no mode, **Range**: 16

James
Mean: 41.8 (1dp), **Median**: 32, **Mode**: 23, **Range**: 76

James has two extreme values that have a big impact on the range

"James is less consistent that Lucy because his scores have a greater range. Lucy performed better on average because her scores have a similar mean and a higher median"

GEOMETRY… Angles in parallel lines and polygons

What do I need to be able to do?

- Basic angles rules and notation
- Angles between parallel lines and a transversal
- Alternate and corresponding angles
- Alternate, corresponding and co-interior angles
- Solve complex problems with angles in parallel lines
- Properties of special quadrilaterals and their diagonals
- Find sides and angles in special quadrilaterals
- Exterior angles of a polygon
- Interior angles of a polygon
- Interior angles in a regular polygon
- Prove simple geometric proofs (E)

Keywords

Tier 2:

Parallel: Straight lines that never meet

Angle: The figure formed by two straight lines meeting (measured in degrees)

Transversal: A line that cuts across two or more other (normally parallel) lines

Sum: Addition (total of all the interior angles added together)

Regular polygon: All the sides have equal length; all the interior angles have equal size.

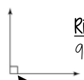
Tier 3:

Polygon: A 2D shape made with straight lines

Isosceles: Two equal size lines and equal size angles (in a triangle or trapezium)

Basic angle rules and notation

Acute Angles
0°< angle <90°

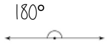


Right Angles
90°

Right angle notation

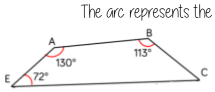
Obtuse
90°< angle <180°

Reflex
180°< angle <360°



Straight Line
180°

The letter in the middle is the angle
The arc represents the part of the angle



Angle Notation: three letters ABC
This is the angle at B = 113 °
Line Notation: two letters EC
The line that joins E to C.

Vertically opposite angles
Equal

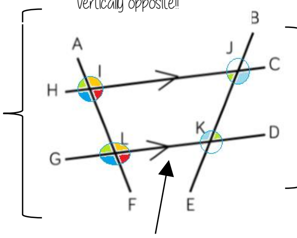
Angles around a point
360 °

Parallel lines

Still remember to look for angles on straight lines, around a point and vertically opposite!!

Lines AF and BE are **transversals** (lines that bisect the parallel lines)

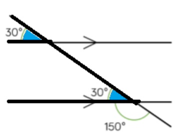
Corresponding angles often identified by their **"F shape"** in position



Alternate angles often identified by their **"Z shape"** in position

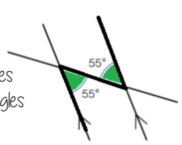
This notation identifies parallel lines

Alternate/ Corresponding angles

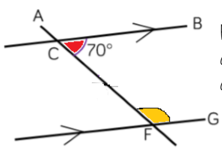


Because alternate angles are equal the highlighted angles are the same size

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles



Because co-interior angles have a sum of 180° the highlighted angle is 110°

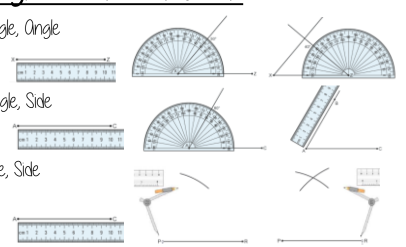
As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

Triangles & Quadrilaterals

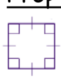
Side, Angle, Angle

Side, Angle, Side

Side, Side, Side



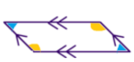
Properties of Quadrilaterals



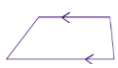
Square
All sides equal size
All angles 90°
Opposite sides are parallel

Rectangle
All angles 90°
Opposite sides are parallel

Rhombus
All sides equal size
Opposite angles are equal



Parallelogram
Opposite sides are parallel
Opposite angles are equal
Co-interior angles

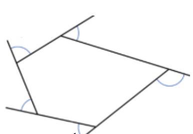


Trapezium
One pair of parallel lines

Kite
No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

Sum of exterior angles

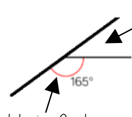
Exterior angles all add up to 360°



Exterior Angles
Are the angle formed from the straight-line extension at the side of the shape

Using exterior angles

Exterior Angle



Interior Angle

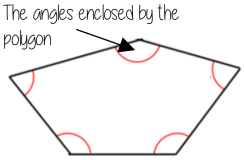
Interior angle + Exterior angle = straight line = 180°
Exterior angle = 180 – 165 = **15°**

Number of sides = 360° ÷ exterior angle
Number of sides = 360 ÷ 15 = 24 sides

Sum of interior angles

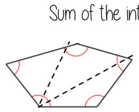
(number of sides – 2) x 180

Interior Angles
The angles enclosed by the polygon



This is an **irregular** polygon – the sides and angles are different sizes

Sum of the interior angles = (5 – 2) x 180

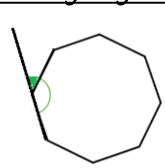


This shape can be made from three triangles
Each triangle has 180°

Sum of the interior angles = 3 x 180
= 540°

Remember this is **all** of the interior angles added together

Missing angles in regular polygons



Exterior angle = 360 ÷ 8 = 45°

Interior angle = $\frac{(8-2) \times 180}{8} = \frac{6 \times 180}{8} = 135°$

Exterior angles in regular polygons = 360° ÷ number of sides

Interior angles in regular polygons = $\frac{(\text{number of sides} - 2) \times 180}{\text{number of sides}}$

PROBABILITY… Tables and Probability

What do I need to be able to do?

- Probability vocabulary
- The probability scale
- Probability of a single event
- Use the sum of probabilities being equal to 1
- Probability experiments
- Sample spaces for 1 or more events
- Probabilities from sample space diagrams
- Two-way tables
- Probabilities from two-way tables
- Frequency trees
- Probabilities from frequency trees

Keywords

Tier 2:

Outcomes: the result of an event that depends on probability.

Probability: the chance that something will happen.

Set: a collection of objects.

Chance: the likelihood of a particular outcome.

Event: the outcome of a probability – a set of possible outcomes.

Biased: a built-in error that makes all values wrong by a certain amount.

Union: Notation '∪' meaning the set made by comparing the elements of two sets.

Identify and represent sets

The **universal set** has this symbol ξ – this means EVERYTHING in the Venn diagram is in this set

A set is a collection of things – you write sets inside curly brackets { }

ξ = {the numbers between 1 and 50 inclusive}

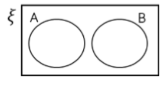
My sets can include every number between 1 and 50 including those numbers

A = {Square numbers}

A = {1, 4, 9, 16, 25, 36, 49}

All the numbers in set A are square number and between 1 and 50

Interpret and create Venn diagrams



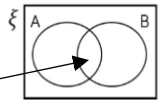
Mutually exclusive sets

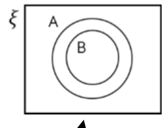
The two sets have nothing in common

No overlap

Union of sets

The two sets have some elements in common – they are placed in the intersection





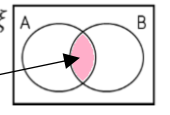
Subset

All of set B is also in Set A so the ellipse fits inside the set

The box

Around the outside of every Venn diagram will be a box. If an element is not part of any set it is placed outside an ellipse but inside the box

Intersection of sets

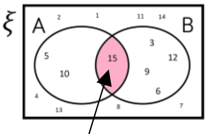


Elements in the intersection are in set A AND set B

The notation for this is $A \cap B$

ξ = {the numbers between 1 and 15 inclusive}

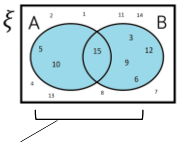
A = {Multiples of 5} B = {Multiples of 3}



The element in $A \cap B$ is 15

In this example there is only one number that is both a multiple of 3 and a multiple of 5 between 1 and 15

Union of sets



ξ = {the numbers between 1 and 15 inclusive}

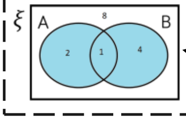
A = {Multiples of 5} B = {Multiples of 3}

Elements in the union could be in set A OR set B

The elements in $A \cup B$ are 5, 10, 15, 3, 9, 6, 12

The notation for this is $A \cup B$

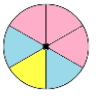
There are 7 elements that are either a multiple of 5 OR a multiple of 3 between 1 and 15



This Venn shows the **number of elements** in each set

Sample space – for single events

A sample space for rolling a six-sided dice is S = {1, 2, 3, 4, 5, 6}



A sample space for this spinner is S = {Pink, Blue, Yellow}

You only need to write each element once in a sample space diagram

- A Sample space represents a possible outcome from an event
- They can be interpreted in a variety of ways because they do not tell you the probability

Probability of a single event

Probability = $\frac{\text{number of times event happens}}{\text{total number of possible outcomes}}$

P(Blue) = $\frac{4}{10}$ = $\frac{2}{5}$

There are 4 blue sectors

There are 10 sectors overall

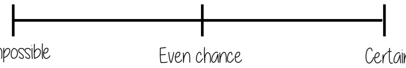
Probability notation P (event)

Probability can be a fraction, decimal or percentage value

$\frac{4}{10}$ = $\frac{40}{100}$ = 0.40 = 40%

Probability is always a value between 0 and 1

The probability scale

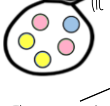


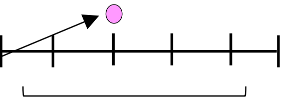
Impossible 0 or 0%

Even chance 0.5, $\frac{1}{2}$ or 50%

Certain 1 or 100%

The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)



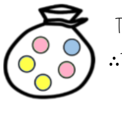


There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Sum of probabilities

Probability is always a value between 0 and 1



The probability of getting a blue ball is $\frac{1}{5}$

∴The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

P(white chocolate) = 1 – 0.15 – 0.35

= 0.5

PROBABILITY... Tables and Probability

Construct sample space diagrams

Sample space diagrams provide a systematic way to display outcomes from events

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

This is the set notation to list the outcomes S =

In between the { } are a;; the possible outcomes

S = { 1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T}

Probability from sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

What is the probability that an outcome has an even number and a tails?

This is the set notation that represents the question P

P (Even number and Tails) = $\frac{3}{12}$

In between the ( ) is the event asked for

There are three even numbers with tails

There are twelve possible outcomes

Numerator: the event

Denominator: the total number of outcomes

Probability from two-way tables

	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

P (Girl walk to school) = $\frac{21}{100}$

The event

The total in the set

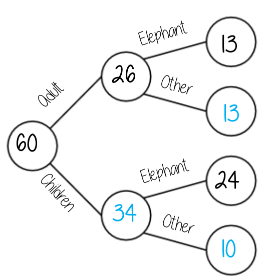
The total number of items

Product Rule

The number of items in event a X The number of items in event b

Probability from frequency trees

60 people visited the zoo one Saturday morning.
26 of them were adults.
13 of the adult's favourite animal was an elephant.
24 of the children's favourite animal was an elephant.



Frequency trees and two-way tables can show the same information

The total columns on two-way tables show the possible denominators

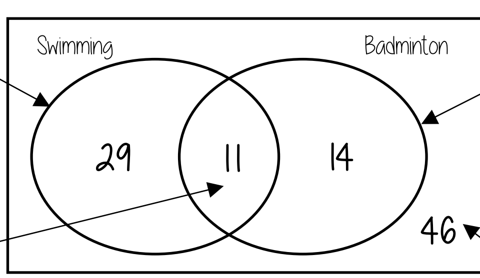
P(adult) = $\frac{26}{60}$

P(Child with favourite animal as elephant) = $\frac{13}{37}$

Probability from Venn diagrams

100 students were questioned if they played badminton or went to swimming club.
40 went swimming, 25 went to badminton and 11 went to both.

This whole curve includes everyone that went swimming.
Because 11 did both we calculate **just** swimming by 40- 11



This whole curve includes everyone that went to badminton.
Because 11 did both we calculate **just** badminton by 25 - 11

P (Just swimming) = $\frac{29}{100}$

The intersection represents both Swimming AND badminton

The number outside represents those that did **neither** badminton or swimming

100 – 29 – 11 – 14

GEOMETRY... Circles

What do I need to be able to do?

- Circle vocabulary
- Pi as a ratio
- Circumference of a circle
- Perimeter of parts of a circle
- Area of a circle
- Area of parts of a circle
- Area and circumference of a circle
- Perimeter of compound shapes with circles
- Perimeter and area of compound shapes with circles

Keywords

Tier 2:

Congruent: Same shape, same size

Area: Space inside a 2D object

Perimeter: Length around the outside of a 2D object

Circumference: Length around the outside of a circle

Pi (π): The ratio of a circle's circumference to its diameter.

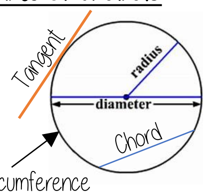
Perpendicular: At an angle of 90° to a given surface

Formula: A mathematical relationship/ rule given in symbols. E.g. b x h = area of rectangle/ square

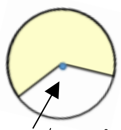
Infinity (∞): A number without a given ending (too great to count to the end of the number) – never ends

Sector: A part of the circle enclosed by two radii and an arc.

Parts of a circle

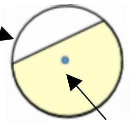


Circumference



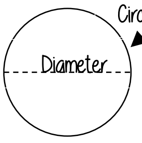
Sector (part of the circle made from two radii)

An arc is a part of the circumference



Segment (part of the circle made from a chord)

Circumference



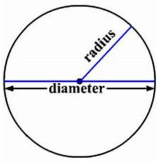
Circumference

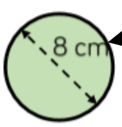
Circumference
π x diameter

Area of a circle (Non-Calculator)

Read the question – leave in terms of π or if $\pi \approx 3$ (provides an estimate for answers)

Area of a circle
π x radius^2



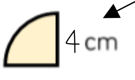


Diameter = 8cm
∴ **Radius** = 4cm

π x radius^2
= π x 4^2
= π x 16
= 16π cm^2

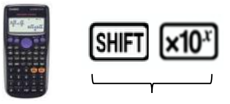
Find the area of one quarter of the circle

Radius = 4cm



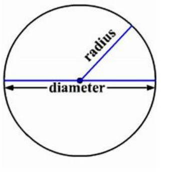
Circle Area = 16π cm^2
Quarter = 4π cm^2

Area of a circle (Calculator)



How to get π symbol on the calculator

Area of a circle
π x radius^2



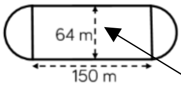
It is important to round your answer suitably – to significant figures or decimal places. This will give you a decimal solution that will go on forever!

Compound shapes including circles

Circumference
π x diameter

Compound shapes are not always area questions. For Perimeter you will need to use the circumference

Spotting diameters and radii



This dimension is also the diameter of the semi circles.

Arc lengths = π x 64
= 64 π

Don't need to halve this because there are 2 ends which make the whole circle

Arc lengths + Straight lengths = total perimeter

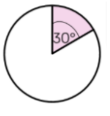
= 64 π + 150 + 150
= (300 + 64 π) m
OR = 501 m

Still remember to split up the compound shape into smaller more manageable individual shapes first

Fractional parts of a circle

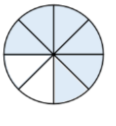
A circle is made up of 360°

Formula to remember:
Area of a circle = πr^2
Circumference of a circle = πd or $2\pi r$



30° represents $\frac{30}{360}$ of a full circle

$\frac{30}{360} = \frac{1}{12}$



$\frac{270}{360}$ of a full circle (in degrees)

$\frac{6}{8}$ of a full circle (in equal parts)

$\frac{3}{4}$ of a full circle

The fraction of the circle is as $\frac{\theta}{360}$

θ represents the degrees in the sector

STATISTICS... Graphs & charts

What do I need to be able to do?

- Pictograms and bar charts
- Vertical line charts
- Draw pie charts
- Interpret pie charts
- Line graphs
- Choose the most appropriate graph or chart
- Compare distributions using graphs
- Misleading graphs and charts

Keywords

Tier 2:

Hypothesis: an idea or question you want to test

Sampling: the group of things you want to use to check your hypothesis

Primary Data: data you collect yourself

Secondary Data: data you source from elsewhere e.g the internet/ newspapers/ local statistics

Discrete Data: numerical data that can only take set values

Continuous Data: numerical data that has an infinite number of values (often seen with height, distance, time)

Spread: the distance/ how spread out/ variation of data

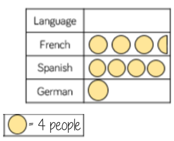
Average: a measure of central tendency – or the typical value of all the data together

Proportion: numerical relationship that compares two things

Pictograms, bar and line charts

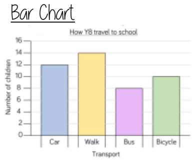
Represents quantitative data

Pictogram



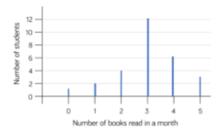
- Need to remember a key
- Visually able to identify mode

Bar Chart



- Gaps between the bars
- Clearly labelled axes
- Scale for the axes
- Title for the bar chart
- Discrete Data

Line Chart



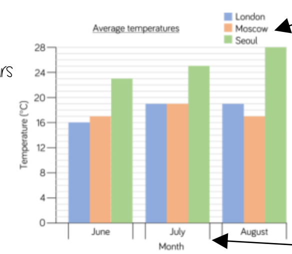
- Gaps between the lines
- Clearly labelled axes
- Scale for the axes
- Discrete Data

Multiple Bar chart

Compares multiple groups of data

- Clearly labelled axes
- Scale for axes
- Comparable data bars drawn next to each other

Key/ Colour code for separate groups of information



Gap between different categories of data

Draw and interpret Pie Charts

Remember a circle has 360°

Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

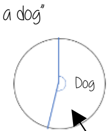
$\frac{32}{60}$ "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192°$

Multiple method

As 60 goes into 360 – 6 times.
Each frequency can be multiplied by 6 to find the degrees (proportion of 360)



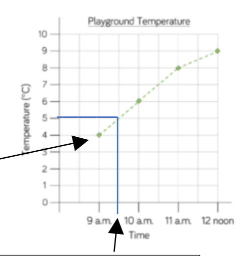
Use a protractor to draw
This is 192°

Represents quantitative, discrete data

Draw and interpret line graphs

- Commonly used to show changing over time
- The points are the recorded information and the lines join the points.

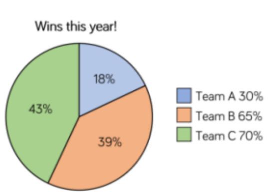
Line graphs do not need to start from 0



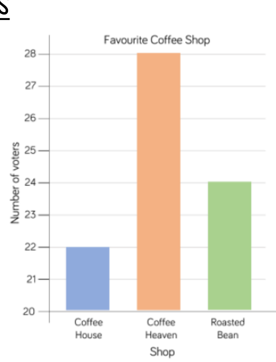
More than one piece of data can be plotted on the same graph to compare data

It is possible to make estimates from the line e.g temperature at 9.30am is 5°C

Misleading graphs and charts



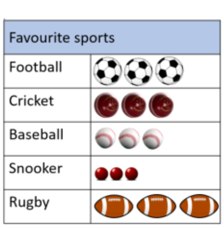
Why are these graphs hard to interpret, and how could they be improved?



The cards show claims made by the owner of Coffee Heaven. Are these claims justified?

Coffee Heaven is the most popular coffee shop in town!

Coffee Heaven is 4 times as popular as other coffee shops!



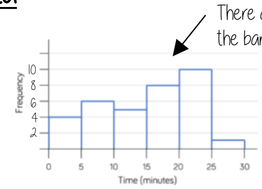
Grouped quantitative data

Time (minutes)	Frequency
$0 \le t < 5$	4
$5 \le t < 10$	6
$10 \le t < 15$	5
$15 \le t < 20$	8
$20 \le t < 25$	10
$25 \le t < 30$	1

"More than or equal to 25 and less than 30 minutes"

The use of inequalities shows that this will be a frequency diagram

This is a frequency diagram
There are no gaps between the bars



Grouping the data is useful if there is a large spread of data to begin with

Find and interpret the range

Difference between the biggest and smallest values

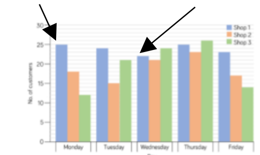
The range is a measure of **spread**

A smaller range means there is less variation in the results – it is more consistent data

A range of 0 means all the data is the same value

Shop 1 highest value

Shop 1 lowest value



Shop 1 has the smallest range – this indicates it has a more consistent flow of customers each week

Range of customers = 25 – 22 = 3 (Shop 1)

ALGEBRA...

Sequences

What do I need to be able to do?

- Generate and describe a sequence given a rule in words
- Generate a sequence given a simple algebraic rule
- nth term of a linear sequence
- Generate a sequence given a complex algebraic rule (E)

Keywords

Tier 2:
Sequence: items or numbers put in a pre-decided order
Term: a single number or variable
Position: the place something is located
Linear: the difference between terms increases or decreases (+ or -) by a constant value each time
Non-linear: the difference between terms increases or decreases in different amounts, or by x or ÷
Difference: the gap between two terms
Arithmetic: a sequence where the difference between the terms is constant
Geometric: a sequence where each term is found by multiplying the previous one by a fixed non zero number

Linear and Non Linear Sequences

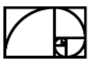
Linear Sequences – increase by addition or subtraction and the same amount each time
Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

Fibonacci Sequence – look out for this type of sequence

0 1 1 2 3 5 8 ...

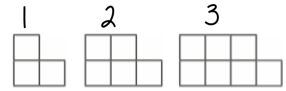
Each term is the sum of the previous two terms.



Sequence in a table and graphically

Position: the place in the sequence

1 2 3



3 5 7

Term: the number or variable
(the number of squares in each image)

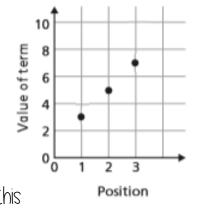
"The **term** in **position** 3 has 7 squares"

In a table

Position	1	2	3
Term	3	5	7

+2 +2

Graphically



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

Sequences from algebraic rules

This is substitution!

$3n + 7$

This will be linear - note the single power of n. The values increase at a constant rate

$3n^2 + 7$

This is not linear as there is a power for n

$2n - 5$ → Substitute the number of the term you are looking for in place of 'n'

eg
1st term = 2 (1) – 5 = -3
2nd term = 2 (2) – 5 = -1
100th term = 2 (100) – 5 = 195

Checking for a term in a sequence

Form an equation

Is 201 in the sequence 3n – 4?

$3n - 4 = 201$

Algebraic rule

Term to check

Solving this will find the position of the term in the sequence. ONLY an integer solution can be in the sequence.

Complex algebraic rules

Misconceptions and comparisons

$2n^2$

2 times whatever n squared is

eg
1st term = 2 x 1^2 = 2
2nd term = 2 x 2^2 = 8
100th term = 2 x 100^2 = 2000

$(2n)^2$

2 times n then square the answer

eg
1st term = $(2 \times 1)^2$ = 4
2nd term = $(2 \times 2)^2$ = 16
100th term = $(2 \times 100)^2$ = 40000

$n(n + 5)$

eg
1st term = 1 (1 + 5) = 6
2nd term = 2 (2 + 5) = 14
100th term = 100 (100 + 5) = 10500

You don't need to expand the expression

Finding the algebraic rule

This is the 4 times table → 4, 8, 12, 16, 20...

4n

7, 11, 15, 19, 22 ← This has the same constant difference – but is 3 more than the original sequence

$4n + 3$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

$4n + 3$