

YEAR 7

KNOWLEDGE ORGANISERS



ALGEBRA...

Sequences

What do I need to be able to do?

- Sequences of diagrams
- Continue number sequences
- Term-to-term rules
- Sequences from a written rule
- Linear and non-linear sequences

Keywords

Tier 2:

Sequence: items or numbers put in a pre-decided order

Position: the place something is located

Rule: instructions that relate two variables

Difference: the gap between two terms

Term: a single number or variable

Linear: the difference between terms increases or decreases by the same value each time

Non-linear: the difference between terms increases or decreases in different amounts

Tier 3:

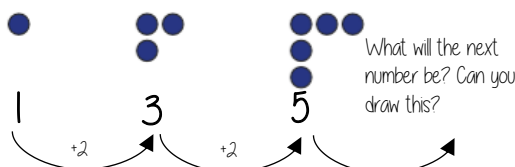
Arithmetic: a sequence where the difference between the terms is constant

Geometric: a sequence where each term is found by multiplying the previous one by a fixed non zero number

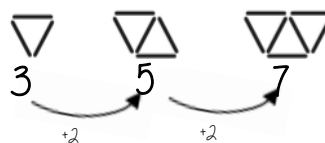


Describe and continue a sequence diagrammatically

Count the number of circles or lines in each image



Predict and check terms



CHECK — draw the next terms



Predictions:

Look at your pattern and consider how it will increase.

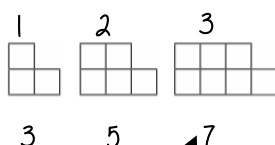
e.g. How many lines in pattern 6?

Prediction - 13

If it is increasing by 2 each time - in 3 more patterns there will be 6 more lines

Sequence in a table and graphically

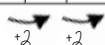
Position: the place in the sequence



Term: the number or variable (the number of squares in each image)

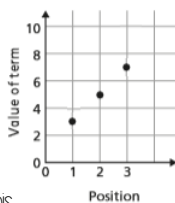
In a table

Position	1	2	3
Term	3	5	7



Because the terms increase by the same addition each time this is **linear** — as seen in the graph

Graphically



"The term in position 3 has 7 squares"

Linear and Non Linear Sequences

Linear Sequences — increase by addition or subtraction and the same amount each time

Non-linear Sequences — do not increase by a constant amount — quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

Fibonacci Sequence — look out for this type of sequence

0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms

Continue Linear Sequences

7, 11, 15, 19...

How do I know this is a linear sequence?

It increases by adding 4 to each term

How many terms do I need to make this conclusion?

At least 4 terms — two terms only shows one difference not if this difference is constant (a common difference)

How do I continue the sequence?

You continue to repeat the same difference through the next positions in the sequence

Continue non-linear Sequences

1, 2, 4, 8, 16 ...

How do I know this is a non-linear sequence?

It increases by multiplying the previous term by 2 — this is a geometric sequence because the constant is multiply by 2

How many terms do I need to make this conclusion?

At least 4 terms — two terms only shows one difference not if this difference is constant (a common difference)

How do I continue the sequence?

You continue to repeat the same difference through the next positions in the sequence

Explain term-to-term rule

How you get from term to term

Try to explain this in full sentences not just with mathematical notation

Use key maths language — doubles, halves, multiply by two, add four to the previous term etc

To explain a whole sequence you need to include a term to begin at...

The next term is found by tripling the previous term
The sequence begins at 4

4, 12, 36, 108...

First term

ALGEBRA. Algebraic notation & substitution

What do I need to be able to do?

- 1-step function machines (number)
- 1-step function machines (algebra)
- Find a function (one step)
- Substitution (one step)
- 2-step function machines (number)
- 2-step function machines (algebra)
- Substitution (two step)

Keywords

Tier 2:

Operation: a mathematical process

Inverse: the operation that undoes what was done by the previous operation (The opposite operation)

Evaluate: work out

Function: a relationship that instructs how to get from an input to an output

Input: the number/ symbol put into a function

Output: the number/ expression that comes out of a function

Substitute: replace one variable with a number or new variable

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

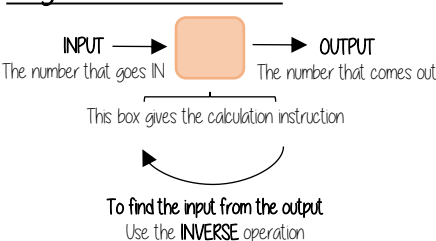
Linear: the difference between terms increases or decreases by the same value each time

Sequence: items or numbers put in a pre-decided order

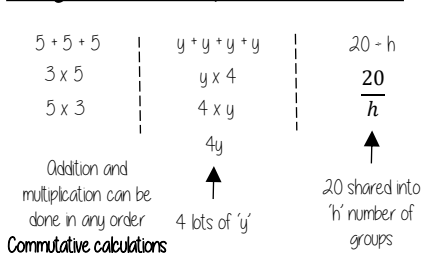
Tier 3:

Commutative: the order of the operations do not matter

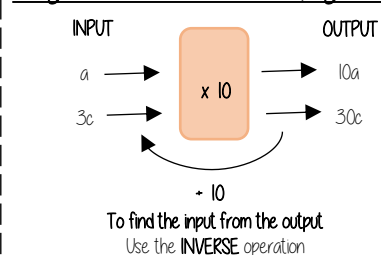
Single function machines



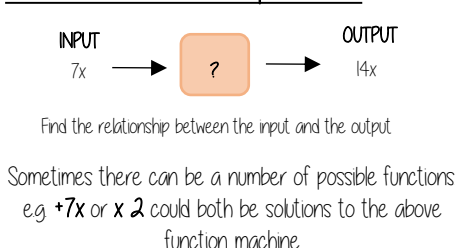
Using letters to represent numbers



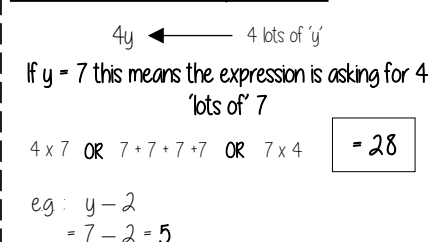
Single function machines (algebra)



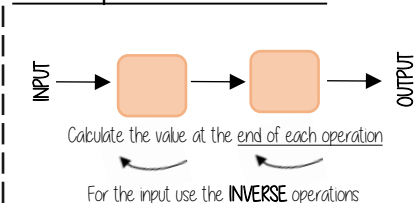
Find functions from expressions



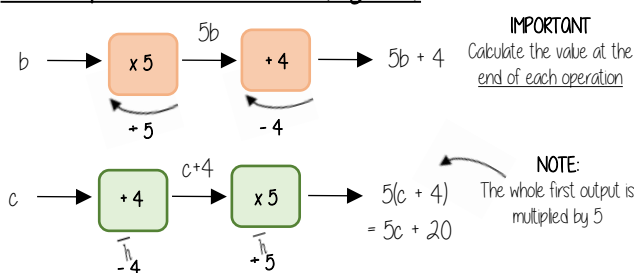
Substitution into expressions



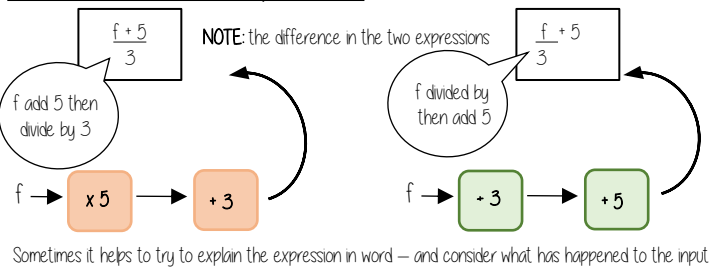
Two step function machines



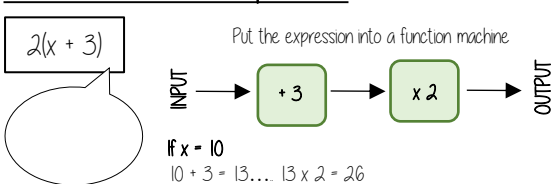
Two step function machines (algebra)



Find functions from expressions



Substitution into an expression



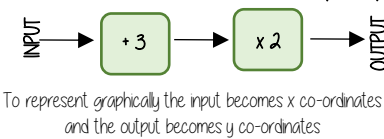
Forming a sequence

INPUT	1	2	3
OUTPUT	8	10	12

The substitution is the 'input' value
The OUTPUT becomes the sequence

Representing functions graphically

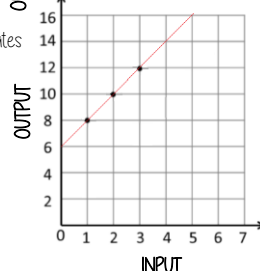
Take the function and generate a sequence $2(x + 3)$



$$y = 2(x + 3)$$

INPUT (x)	1	2	3
OUTPUT (y)	8	10	12

This becomes a co-ordinate pair (2, 10) to plot on a graph



Not all graphs will be linear only those with an integer value for x.
Powers and fractions generate differently shaped graphs

NOTE: Because this is a linear graph you can predict other values

ALGEBRA...

Expressions and equations

What do I need to be able to do?

- Like and unlike terms
- Collect like terms
- Solve 1-step equations (+/-)
- Solve 1-step equations (x/+)
- Solve any 1-step equation

Keywords

Tier 2:

Equals: represented by '=' symbol — means the same

Equality: two expressions that have the same value

Solution: the set or value that satisfies the equation

Equation: a mathematical statement that two things are equal

Inverse: the operation that undoes what was done by the previous operation (The opposite operation)

Like: variables that are the same are 'like'

Term: a single number or variable

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Tier 3:

Coefficient: a multiplicative factor in front of a variable e.g. $5x$ (5 is the coefficient, x is the variable)

Solve: to find the solution

Equality

$$2 + 14 = 5 + 5 + 6$$

16

16

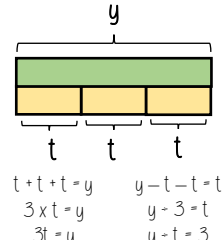
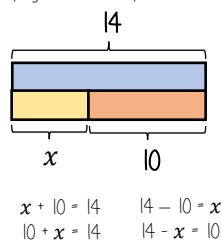
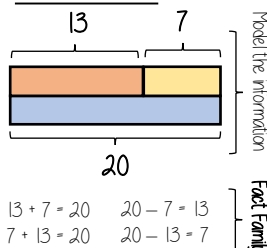
"Is equal to"

Saying it out loud sometimes helps you to understand equality

The sum on the left has the same result as the sum on the right

Fact Families

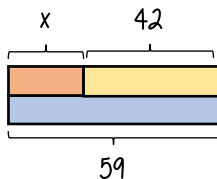
Use a bar model to display the relationships between terms and numbers



Solve one step equations (+/-)

There is more to this than just spotting the answer

$$x + 42 = 59$$



Don't forget you know how to use function machines



Solve one step equations (x/+)

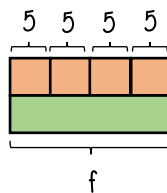
$$\frac{f}{4} = 5$$

$$f - 4 = 5$$

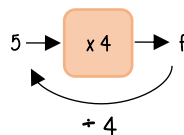
$$f - 5 = 4$$

$$5 \times 4 = f$$

$$4 \times 5 = f$$



Don't forget you know how to use function machines



Like and unlike terms

Like terms are those whose variables are the same

♥ and 3♥ are like terms
 the variable is the same

★ and 3♥ are unlike terms
 the variables are NOT the same

Examples and non-examples

Like terms

y , $7y$
 $2x^2$, x^2
 ab , $10ba$
 5 , -2

Un-like terms

y , $7x$
 $2x^2$, $2c^2$
 ab , $10a$
 5 , $-2t$

Note here ab and ba are commutative operations, so are still like terms

Equivalence

Check equivalence by substitution

e.g. $m = 10$

$$5m$$

$$5 \times 10$$

$$= 50$$

$$2 \times 2m$$

$$2 \times (2 \times 10)$$

$$= 2 \times 20$$

$$= 40$$

$$7m - 3m$$

$$(7 \times 10) - (3 \times 10)$$

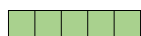
$$= 70 - 30$$

$$= 40$$

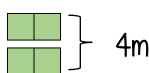
Equivalent expressions

Repeat this with various values for m to check

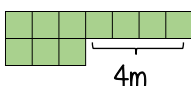
$$5m$$



$$2 \times 2m$$



$$7m - 3m$$



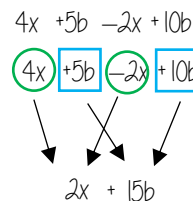
Collecting like terms $\equiv$ symbol

The $\equiv$ symbol means equivalent to

It is used to identify equivalent expressions

Collecting like terms

Only like terms can be combined



Common misconceptions

$$2x + 3x^2 + 4x \equiv 6x + 3x^2$$

Although they both have the x variable x^2 and x terms are unlike terms so cannot be collected

NUMBER...

Place value & ordering

What do I need to be able to do?

- Read and write integers to 10 000
- Understand the place value of a digit in an integer to 10 000
- Compare integers to 10 000
- Order integers to 10 000
- Work out intervals on a number line
- Position integers on a number line
- Place value for decimals
- Compare and order decimals

Keywords

Tier 2:

Approximate: To estimate a number, amount or total often using rounding of numbers to make them easier to calculate with

Interval: between two points or values

Negative: Any number less than zero, written with a minus sign

Place holder: We use 0 as a place holder to show that there are none of a particular place in a number

Standard (index) Form: A system of writing very big or very small numbers

Base: The number that gets multiplied by a power

Power: The exponent — or the number that tells you how many times to use the number in multiplication

Exponent: The power — or the number that tells you how many times to use the number in multiplication

Tier 3:

Integer: a whole number that is positive or negative

Place value: The value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

Integer Place Value

Billions			Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O
		3	1	4	8	0	3	3	0	2	9

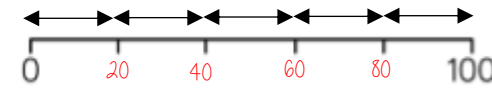
Placeholder

Three billion, one hundred and forty eight million, thirty three thousand and twenty nine

1 billion 1,000,000,000

1 million 1,000,000

Intervals on a number line

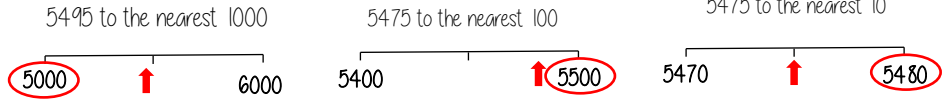


Divide the difference by the number of intervals (gaps).

E.g. $100 \div 5 = 20$

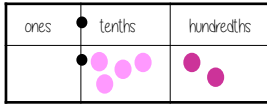
Rounding to the nearest power of ten

If the number is halfway between we "round up"



Decimals

We say "nought point five two"



0 ones, 5 tenths and 2 hundredths

$$0 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.01 + 0.01$$

$$= 0 + 0.5 + 0.02$$

$$= 0.52$$

Five tenths and two hundredths

Compare integers using $<$, $>$, $=$, $\neq$

$<$ less than

$>$ greater than

$=$ equal to

$\neq$ not equal to

Two and a half million $=$ 2 500 000

300 000 000 $=$ Three billion

Six thousand and eighty $<$ 68 000

Positive powers of 10

1 billion = 1 000 000 000

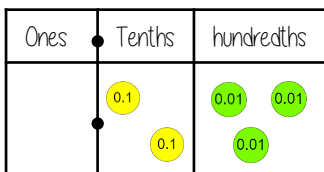
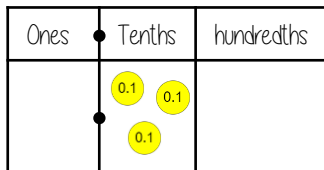
$$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$$

Addition rule for indices $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices $10^a \div 10^b = 10^{a-b}$

Comparing decimals

Which the largest of 0.3 and 0.23?



$$0.3 > 0.23$$

"There are more counters in the furthest column to the left"

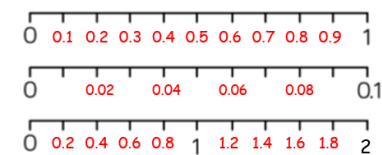
0.30
0.23

Comparing the values both with the same number of decimal places is another way to compare the number of tenths and hundredths

Decimal intervals on a number line

One whole split into 10 parts makes tenths = 0.1

One tenth split into 10 parts makes hundredths = 0.01



Standard form with numbers > 1

Any number between 1 and less than 10

$$A \times 10^n$$

Any integer

Example

$$3.2 \times 10^4$$

$$= 3.2 \times 10 \times 10 \times 10 \times 10$$

$$= 32000$$

Non-example

$$(0.8) \times 10^4$$

$$5.3 \times 10^{07}$$

Negative powers of 10

0.001

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
0	0	0	0	1

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Numbers between 0 and 1

$$0.054 = 5.4 \times 10^{-2}$$

1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^0	10^{-1}	10^{-2}	10^{-3}
0	0	5	4

A negative power does not mean a negative answer — it means a number closer to 0

NUMBER...

Four Operations

What do I need to be able to do?

- Add and subtract integers
- Add and subtract decimals
- Multiply and divide by 10, 100 and 1000
- Multiply by 0.1 and 0.01 (E)
- Multiply integers
- Divide integers
- Multiply decimals
- Divide decimals by integers
- Divide by a decimal (E)
- Order of operations

Keywords

Tier 2:

Inverse: the operation that undoes what was done by the previous operation (The opposite operation)

Placeholder: a number that occupies a position to give value

Quotient: the result of a division

Dividend: the number being divided

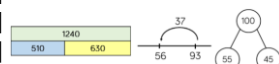
Tier 3:

Divisor: the number we divide by

Commutative: changing the order of the operations does not change the result

Associative: when you add or multiply you can do so regardless of how the numbers are grouped

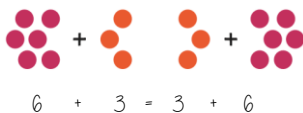
Addition/ Subtraction with integers



Modelling methods for addition/ subtraction

- Bar models
- Number lines
- Part/ Whole diagrams

Addition is commutative



The order of addition does not change the result

Subtraction the order has to stay the same

$$360 - 147 = 360 - 100 - 40 - 7$$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction
- Show your relationships by writing fact families

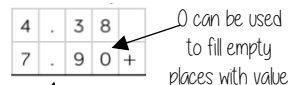
Formal written methods

	H	T	O
	1	8	7
+	5	4	2

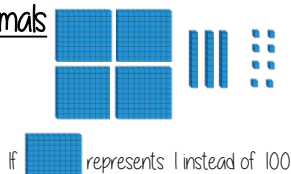
	H	T	O
	4	2	7
-	2	4	9

Remember the place value of each column
You may need to move 10 ones to the ones column to be able to subtract

Addition/ Subtraction with decimals



The decimal place acts as the placeholder and aligns the other values

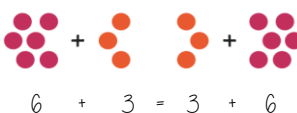


$$5.43 + \frac{8}{10}$$

Revisit Fraction - Decimal equivalence
 $5.43 + 0.8$

Mental methods for addition/ subtraction

Addition is commutative



The order of addition does not change the result

Subtraction the order has to stay the same

$$360 - 147 = 360 - 100 - 40 - 7$$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction

Multiplication methods

	H	T	O
×	1	8	7
			9

Long multiplication (column)

Grid method

1	8	7
1	8	7
1	8	7
1	8	7
1	8	7
1	8	7
1	8	7
1	8	7
1	8	7
1	8	7

Repeated addition

Less effective method especially for bigger multiplication

Multiplication with decimals

Perform multiplications as integers
eg $0.2 \times 0.3 \rightarrow 2 \times 3$

Make adjustments to your answer to match the question: $0.2 \times 10 = 2$
 $0.3 \times 10 = 3$

Therefore $6 \div 100 = 0.06$

Mental methods for multiplication/ division

Multiplication is commutative



$$2 \times 4 = 4 \times 2$$

The order of multiplication does not change the result

Partitioning can help multiplication

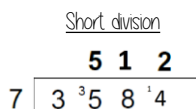
$$\begin{aligned} 24 \times 6 &= 20 \times 6 + 4 \times 6 \\ &= 120 + 24 \\ &= 144 \end{aligned}$$

Division is not associative

Chunking the division can help $4000 \div 25$
"How many 25's in 100" then how many chunks of that in 4000

Division methods

$$3584 \div 7 = 512$$



Complex division

$$\div 24 = \div 6 \div 4$$

Break up the divisor using factors

Division with decimals

The placeholder in division methods is essential - the decimal lines up on the dividend and the quotient

$$24 \div 0.02 \rightarrow 24 \div 0.2 \rightarrow 240 \div 2$$

All give the same solution as represent the same proportion
Multiply the values in proportion until the divisor becomes an integer

Mental methods for decimals

Multiplying by a decimal < 1 will make the original value smaller eg $\times 0.1 = \div 10$

Methods for multiplication 12×0.03

$$\begin{aligned} 12 \times 3 &= 36 \\ 12 \times 3 &= 36 \\ 12 \times 0.3 &= 3.6 \\ 12 \times 0.03 &= 0.36 \end{aligned}$$

Methods for addition $2.3 + 2.4$

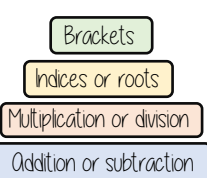
$$\begin{aligned} 2 + 2 &= 4 \\ 0.3 + 0.4 &= 0.7 \\ 4 + 0.7 &= 4.7 \end{aligned}$$

Methods for division $15 \div 0.05$

Multiply by powers of 10 until the divisor becomes an integer

$$\begin{aligned} 1.5 \div 0.05 &\times 100 \rightarrow 150 \div 5 = 30 \end{aligned}$$

Order of operations

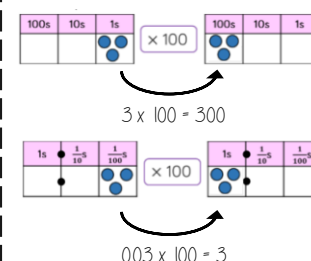


If you have multiple operations from the same tier work from left to right

$$\text{eg } 10 - 3 + 5 \rightarrow 10 - 3 \rightarrow 7 + 5$$

$$\begin{aligned} 6 \times 4 &+ 8 \times 2 \\ 24 &+ 16 = 40 \end{aligned}$$

Multiply/ Divide by powers of 10



Repeated multiplication and division by powers of 10 is commutative

$$\div 10 \text{ then } \div 10 \rightarrow \div 100$$

STATISTICS...

Averages & Range

What do I need to be able to do?

- Mode
- Mean
- Median
- Range
- Solve problems with averages and range

Keywords

Tier 2:

Mode: This is the number OR the item that occurs the most

Median: A measure of central tendency (middle, average) found by putting all the data values in order and finding the middle value of the list

Mean: A measure of average to find the central tendency

Range: The difference between the largest and smallest numbers in a set

Mean, Median, Mode

The Mean

A measure of average to find the central tendency... a typical value that represents the data

24, 8, 4, 11, 8

Find the sum of the data (add the values) 55

Divide the overall total by how many pieces of data you have $55 \div 5$

Mean = 11

The Median

The value in the centre (in the middle) of the data

24, 8, 4, 11, 8

Put the data in order

4, 8, 8, 11, 24

Find the value in the middle 4, 8, 8, 11, 24

Median = 8

NOTE: If there is no single middle value find the mean of the two numbers left

The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8

This can still be easier if the data is ordered first

4, 8, 8, 11, 24

Mode = 8

Range Spread of the values

Difference between the biggest and smallest

3, 9, 8, 12

Range: Biggest value - Smallest value $12 - 3 = 9$ Range = 9

NUMBER...

Rounding & Estimation

What do I need to be able to do?

- Round to 1 significant figure
- Round to 2 or more significant figures
- Estimate answers to calculations
- Solve problems with estimation
- Understand and use error interval notation (E)

Keywords

Tier 2:

Approximate: To estimate a number, amount or total often using rounding of numbers to make them easier to calculate with

Significant figure: A digit that gives meaning to a number. The most significant digit (figure) in an integer is the number on the left. The most significant digit in a decimal is the first non-zero number after the decimal point

Interval: between two points or values

Tier 3:

Place value: The value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

Round to powers of 10 and 1 sig figure

If the number is halfway between we "round up"

5495 to the nearest 1000

5000

5475 to the nearest 100

5500

5475 to the nearest 10

5480

370 to 1 significant figure is 400

37 to 1 significant figure is 40

3.7 to 1 significant figure is 4

0.37 to 1 significant figure is 0.4

0.00037 to 1 significant figure is 0.0004

Round to the first non-zero number

Estimate the calculation

Round to 1 significant figure to estimate

$$4.2 + 6.7 \approx 4 + 7 \approx 11$$

This is an **overestimate** because the 6.7 was rounded up more

The equal sign changes to show it is an estimation

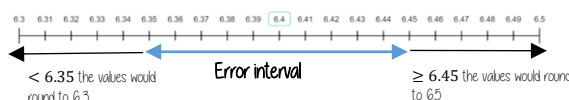
$$21.4 \times 3.1 \approx 20 \times 3 \approx 60$$

This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths - it helps you identify calculation errors.

Error Intervals

A width w has been rounded to 64cm correct to 1dp



The error interval

$$6.35 \leq w < 6.45$$

Any value within these limits would round to 6.4 to 1dp

STATISTICS...

Graphing Data

What do I need to be able to do?

- Pictograms
- Bar charts
- Dual bar charts
- Composite bar charts
- Coordinates in the first quadrant
- Scatter graphs
- Correlation
- Lines of best fit
- Time series graphs
- Non-linear relationships

Keywords

Tier 2:

Primary Data: data you collect yourself

Secondary Data: data you source from elsewhere e.g. the internet/ newspapers/ local statistics

Discrete Data: numerical data that can only take set values

Continuous Data: numerical data that has an infinite number of values (often seen with height, distance, time)

Relationship: the link between two variables (items) E.g. Between sunny days and ice cream sales

Correlation: the mathematical definition for the type of relationship.

Line of best fit: a straight line on a graph that represents the data on a scatter graph

Outlier: a point that lies outside the trend of graph

Quadrant: four quarters of the coordinate plane.

Coordinate: a set of values that show an exact position

Pictograms, bar and line charts

Pictogram

Language	
French	4 circles
Spanish	3 circles
German	1 circle

1 circle = 4 people

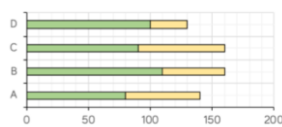
- Need to remember a key
- Visually able to identify mode

Bar Chart



- Gaps between the bars
- Clearly labelled axes
- Scale for the axes
- Title for the bar chart
- Discrete Data

Composite bar charts

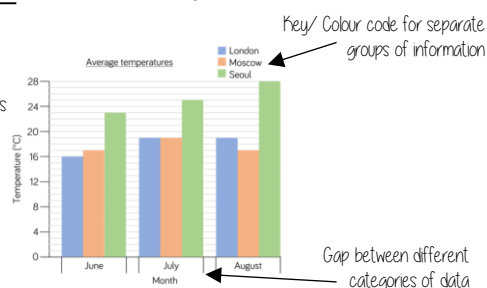


- Categories clearly indicated
- Compare the bars green compared to yellow
- The size of each bar is the frequency
- Overall total easily comparable

Dual Bar chart

Compares multiple groups of data

- Clearly labelled axes
- Scale for axes
- Comparable data bars drawn next to each other

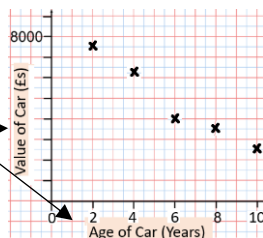


Draw and interpret a scatter graph

Age of Car (Years)	2	4	6	8	10
Value of Car (£)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship

Oil axes should be labelled

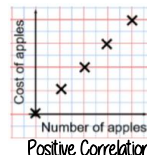


The axis should fit all the values on and be equally spread out

"This scatter graph shows as the age of a car increases the value decreases"

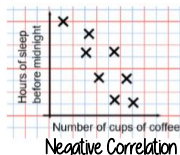
The link between the data can be explained verbally

Linear Correlation



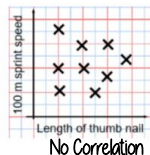
Positive Correlation

As one variable increases so does the other variable



Negative Correlation

As one variable increases the other variable decreases



No Correlation

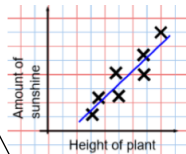
There is no relationship between the two variables

The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph

Things to know:

- The line of best fit **DOES NOT** need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph



It is only an estimate because the line is designed to be an average representation of the data

It is always a **straight line**.

Using a line of best fit

Interpolation is using the line of best fit to estimate values inside our data point.

e.g. 40 hours revising predicts a percentage of 45.

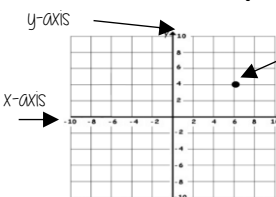


Extrapolation is where we use our line of best fit to predict information outside of our data

This is not always useful — in this example you cannot score more than 100%. So revising for longer can not be estimated

This point is an 'outlier' It is an outlier because it doesn't fit this model and stands apart from the data

Coordinates in four quadrants



Coordinate (x, y) (6, 4)

From the origin this coordinate is 6 places along the positive x axis and 4 places up the positive y axis

(0, a)

Will be always be a point on the y axis (a can be any number)

(a, 0)

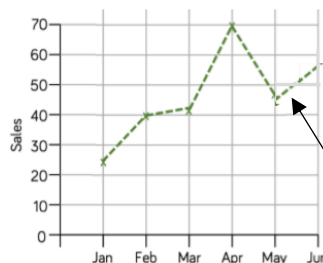
Will be always be a point on the x axis (a can be any number)

Always the position on the x axis first

Always the position on the y axis second

Time-Series

This time-series graph shows the total number of car sales in £1000 over time



Look for general trends in the data. Some data shows a clear increase or a clear decrease over time.

Readings in-between points are estimates (on the dotted lines). You can use them to make assumptions

NUMBER... Fractions, decimals & percentages

What do I need to be able to do?

- Represent tenths and hundredths
- Number lines with fractions and decimals
- Tenths, hundredths, fifths and quarters
- Eighths and thousandths
- Understand percentages
- Convert simple fractions, decimals and percentages
- Fractions as diagrams
- Fractions on a number line
- Equivalent fractions
- Fractions as division
- Convert fractions, decimals and percentages
- Fractions, decimals and percentages greater than 1 (E)

Keywords

Tier 2:

Fraction: how many parts of a whole we have

Decimal: a number with a decimal point used to separate ones, tenths, hundredths etc.

Percentage: a proportion of a whole represented as a number between 0 and 100

Placeholder: a number that occupies a position to give value

Interval: a range between two numbers

Tenth: one whole split into 10 equal parts

Hundredth: one whole split into 100 equal parts

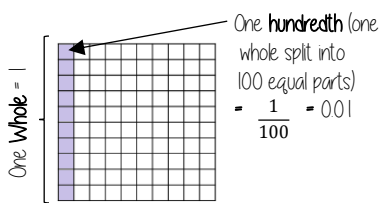
Sector: a part of a circle between two radius (often referred to as looking like a piece of pie)

Recurring: a decimal that repeats in a given pattern

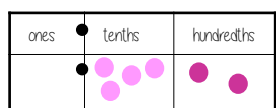
Tier 3:

Place value: the numerical value that a digit has decided by its position in the number

Tenths and hundredths

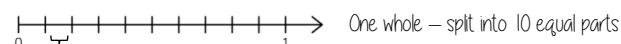


One tenth (one whole split into 10 equal parts) = $\frac{1}{10} = 0.1$

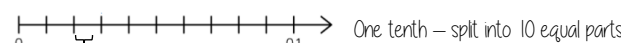


0 ones, 5 tenths and 2 hundredths
 $0 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.01 + 0.01$
 $= 0 + 0.5 + 0.02$
 $= 0.52$

On a number line

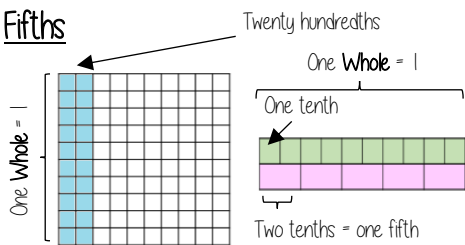


One tenth = $\frac{1}{10} = 0.1$

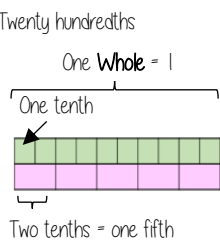


One hundredth = $\frac{1}{100} = 0.01$

Fifths



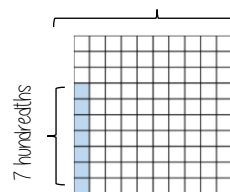
One fifth (one whole split into 5 equal parts) = $\frac{1}{5} = 0.2$



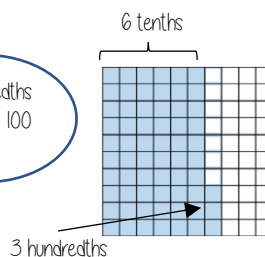
Two tenths = one fifth

Percentages on a hundred grid

100% = a whole = 100 hundredths

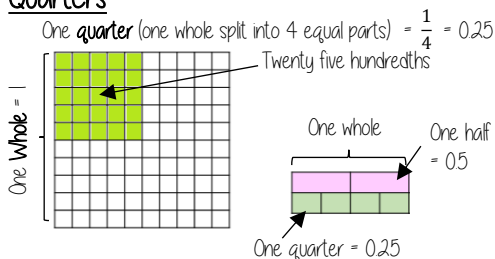


7 hundredths
 7 out of 100
 7%

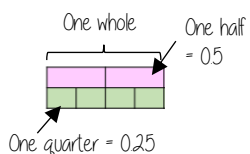


6 tenths and 3 hundredths
 63 hundredths
 63%

Quarters



One quarter (one whole split into 4 equal parts) = $\frac{1}{4} = 0.25$



One quarter = 0.25

Simple pie charts



A pie chart has 360° so all FDP calculations are out of 360

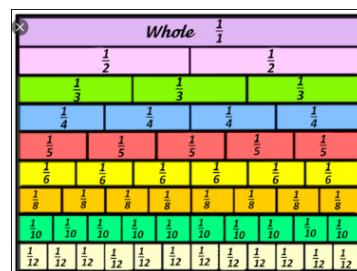
Split into 10 parts
 $= 10\% = 36^\circ$

Split into 2 parts
 $= 50\% = 180^\circ$

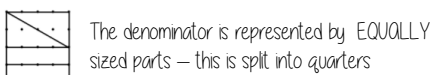
Split into 5 parts
 $= 20\% = 72^\circ$

Equivalent fractions

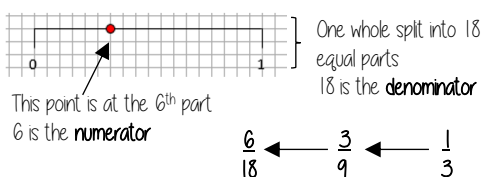
Represent equivalence with fraction walls



Fractions - on a diagram



Fractions - on a number line



This point is at the 6th part
 6 is the numerator

$\frac{6}{18} \leftarrow \frac{3}{9} \leftarrow \frac{1}{3}$

Convert FDP

$\frac{70}{100}$

Using a calculator

$\frac{70}{100}$

This will give you the answer in the simplest form

This also means
 $70 \div 100$

70 out of 100
 squares
 70 'hundredths'
 $= 7 \text{ 'tenths'}$
 0.7



70 hundredths
 $= 70\%$

$\frac{70}{100}$

Convert to a decimal

$\times 100$ converts to a percentage

Be careful of recurring decimals
 eg $\frac{1}{3} = 0.3333333$
 $\frac{1}{3} = 0.\dot{3}$
 The dot above the 3

NUMBER...

Directed Number

What do I need to be able to do?

- Directed number and number lines
- Compare and order directed numbers
- Calculations that cross zero
- Directed number and zero pairs
- Add directed numbers
- Subtract directed numbers
- Multiply and divide directed numbers
- Order of operations with directed numbers
- Use a calculator with directed numbers

Keywords

Tier 2:

Subtract: taking away one number from another.

Negative: a value less than zero.

Product: multiply terms

Inverse: the opposite function

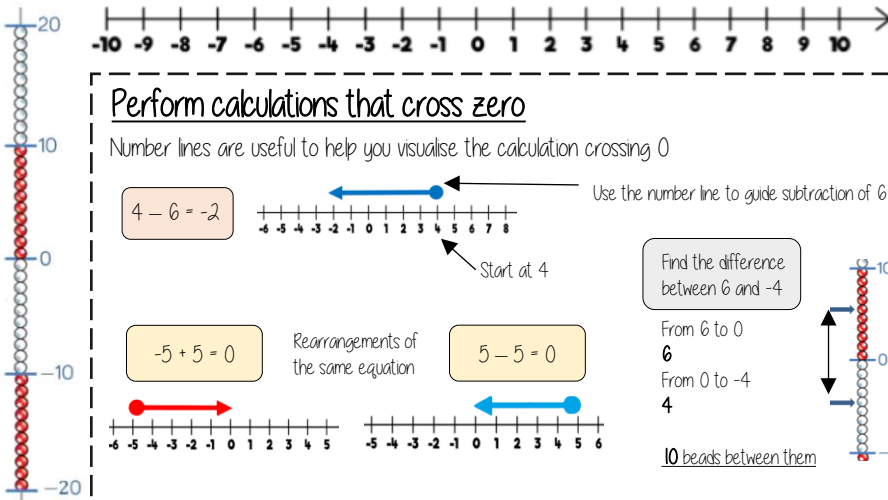
Square: a term multiplied by itself.

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

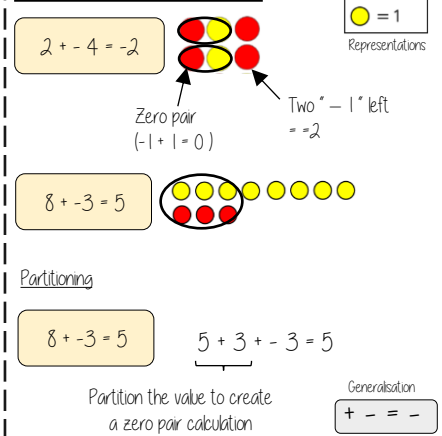
Tier 3:

Commutative: changing the order of the operations does not change the result

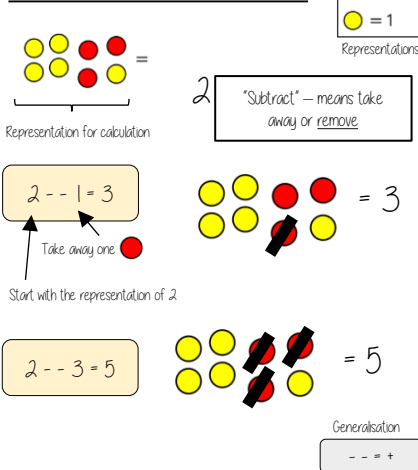
Square root: a square root of a number is a number when multiplied by itself gives the value (symbol $\sqrt{\quad}$)



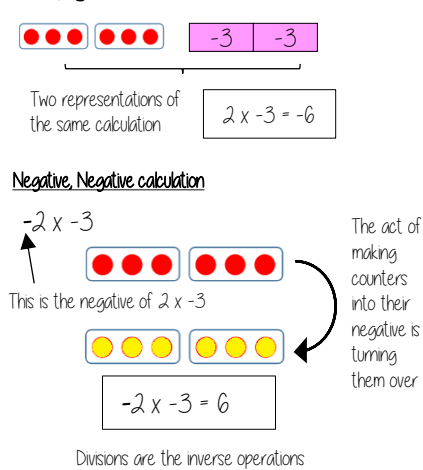
Add directed numbers



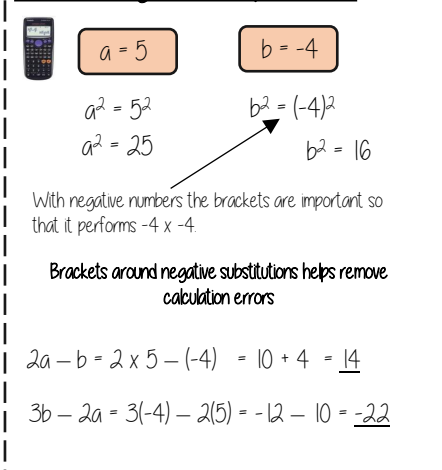
Subtract directed numbers



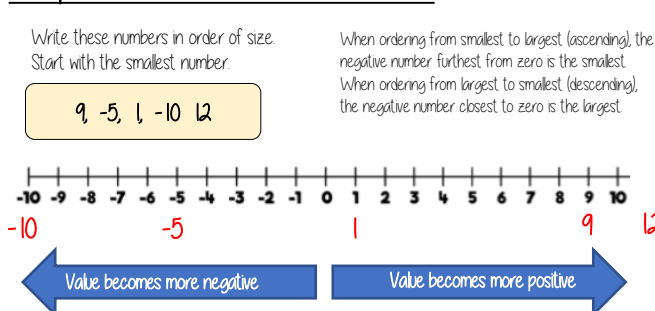
Multiply/ Divide directed numbers



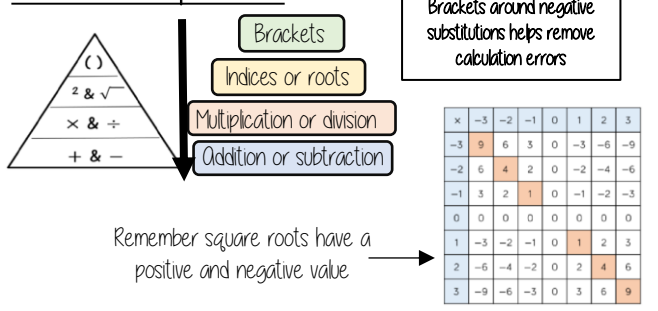
Evaluate algebraic expressions



Compare and order directed numbers



Use order of operations



NUMBER..Fractions & percentages of amounts

What do I need to be able to do?

- Fraction of an amount
- Use a fraction to find the whole
- Percentage of an amount (non-calculator)
- Percentage of an amount (calculator)
- Percentage increase and decrease
- Use a percentage to find the whole (E)
- Solve problems with fractions and percentages greater than 1 (E)

Keywords

Tier 2:

Fraction: how many parts of a whole we have

Equivalent: of equal value

Whole: a number with no fractional or decimal part

Percentage: parts per 100 (uses the % symbol)

Convert: change into an equivalent representation, often fraction to decimal to a percentage cycle

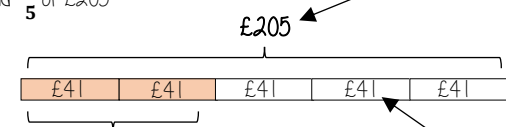
Tier 3:

Place Value: the value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

Fraction of a given amount

Find $\frac{2}{5}$ of £205

The bar represents the whole amount

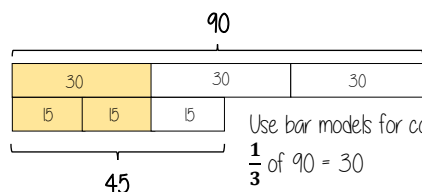


2 out of the 5 equal parts

$$2 \times £41 = \underline{£82}$$

$$£205 \div 5 = £41$$

Each part of the bar model represents £41



Use bar models for comparisons

$$\frac{1}{3} \text{ of } 90 = 30$$

$$\frac{2}{3} \text{ of } 45 = 30$$

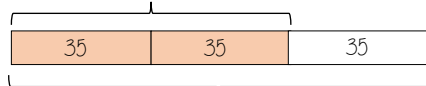
$$\therefore \frac{1}{3} \text{ of } 90 = \frac{2}{3} \text{ of } 45$$

Use a fraction of amount

$\frac{2}{3}$ of a value is 70. What is the whole number?

$$70 \div 2 = 35$$

Each part of the bar model represents 35



$$35 \times 3 = 105$$

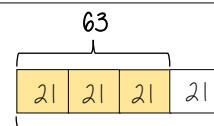
The whole number is 105

The wording of the question is important to setting up the bar model

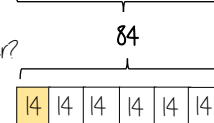
$\frac{3}{4}$ of a number is 63.

What is $\frac{1}{6}$ of the number?

$$= 14$$



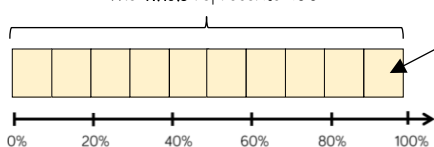
Find the whole



Use the whole to find a given part

Find the percentage of an amount (Mental methods)

The whole represents 100%



$10\% = \frac{1}{10}$ of the whole

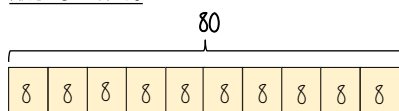
$$10\% = \frac{1}{10} \text{ of the whole}$$

$$50\% = \frac{5}{10} = \frac{1}{2} \text{ of the whole}$$

$$20\% = \frac{2}{10} = \frac{1}{5} \text{ of the whole}$$

$$5\% = \frac{1}{20} \text{ of the whole}$$

Find 65% of 80



Method 1

$$\begin{aligned} 65\% &= 10\% \times 6 + 5\% \\ &= (8 \times 6) + 4 \\ &= 52 \end{aligned}$$

Method 2

$$\begin{aligned} 65\% &= 50\% + 10\% + 5\% \\ &= 40 + 8 + 4 \\ &= 52 \end{aligned}$$

For bigger percentages it is sometimes easier to take away from 100%

Find the percentage of an amount (Calculator methods)



Using a multiplier

Find 65% of 80

Fraction, decimal, percentage conversion

$$65\% = \frac{65}{100} = 0.65$$

The multiplier

$$0.65 \times 80 = \underline{52}$$

Using the percent button

Find 65% of 80

This brings up the % button on screen
You will see 65%

Type 65

Press **SHIFT** **C** **(%)**

Press **x** 80 and then press =

"of" can represent 'x' in calculator methods

You can also use the calculator to support non calculator methods and find 1% or 10% then add percentages together

GEOMETRY...

Perimeter & Area

What do I need to be able to do?

- Convert metric units of length
- 2 Perimeter of a polygon
- Perimeter of a compound shape
- Area of rectangles and parallelograms
- Area of a triangle
- Area of a trapezium
- Solve problems with perimeter and area
- Form expressions with perimeter and area (E)

Keywords

Tier 2:

Milli: prefix meaning one thousandth

Centi: prefix meaning one hundredth

Kilo: prefix meaning multiply by 1000

Congruent: The same

Area: Space inside a 2D object

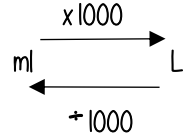
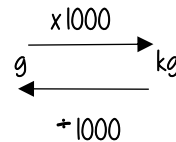
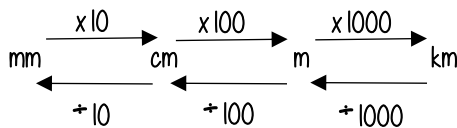
Perimeter: Length around the outside of a 2D object

Perpendicular: At an angle of 90° to a given surface

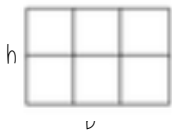
Formula: A mathematical relationship/ rule given in symbols. Eg $b \times h$ = area of rectangle/ square

Metric conversions

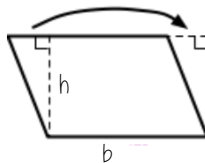
Useful Conversions



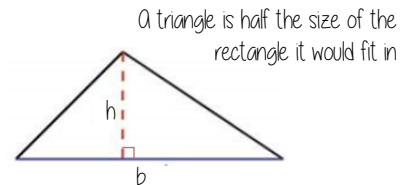
Area – rectangles, triangles, parallelograms



Rectangle
Base x Height



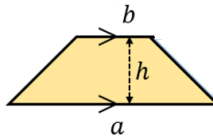
Parallelogram/ Rhombus
Base x Perpendicular height



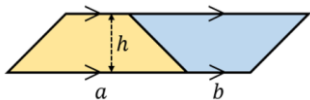
Triangle
 $\frac{1}{2} \times \text{Base} \times \text{Perpendicular height}$

Area of a trapezium

Area of a trapezium
 $\frac{(a+b) \times h}{2}$



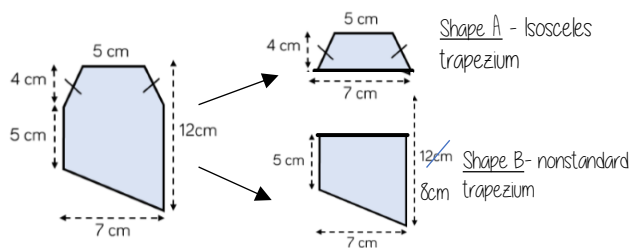
Why?



- Two congruent trapeziums make a parallelogram
- New length $(a+b) \times \text{height}$
- Divide by 2 to find area of one

Compound shapes

To find the area compound shapes often need splitting into more manageable shapes first. Identify the shapes and missing sides etc. first.

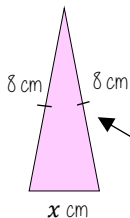


Shape A + Shape B = total area

$$\frac{(5+7) \times 4}{2} + \frac{(5+7) \times 8}{2} = 24 + 45.5 = 69.5 \text{ cm}^2$$

Units

Solve problems with perimeter



Isosceles
Triangle
notation

Perimeter is the length around the outside of a polygon

The triangle has a perimeter of 25cm
Find the length of x

$$8\text{cm} + 8\text{cm} + x\text{cm} = 25\text{cm}$$

$$16\text{cm} + x\text{cm} = 25\text{cm}$$

$$x\text{cm} = 9\text{cm}$$

Forming expressions

More than – ADD

Less than/ difference – SUBTRACT

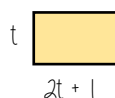
eg 4 more than $t \longrightarrow t + 4$

8 less than $k \longrightarrow k - 8$

Form expressions with perimeter and area

For unknown variables, a letter is normally used in its place

Only similar terms can be grouped together



eg Find the perimeter of this shape
(Perimeter = length around outside of shape)

$$2t + 1$$

$$t + 2t + 1 + t + 2t + 1 \longrightarrow 6t + 2$$

NUMBER...

Speed, distance & time

What do I need to be able to do?

- Convert between milliseconds, seconds, minutes and hour
- Convert between hours, days and year
- Fractions of time
- Solve problems with tables and timetables
- Solve problems with time and the calendar
- Calculate speed
- Calculate time and distance
- Solve problems with speed, distance and time
- Interpret distance-time graphs
- Draw distance-time graphs
- Calculate speed from a distance-time graph (E)

Keywords

Tier 2:

Length: the distance from one point to another

Convert: change a value or expression from one value to another

Mass: a measure of how much matter is in an object. Commonly measured by weight.

Origin: the coordinate (0, 0)

Volume: the amount of 3D space a shape takes up

Substitute: putting numbers where letters are – replacing numbers into a formula

Compound Units: units which require two types of measurement

Time

Convert 34 days to hours

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
52 weeks = 1 year

1 day = 24 hours, so

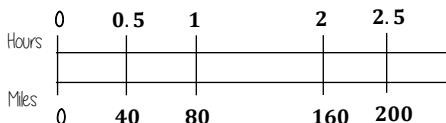
34 days is 34×24
= 816 hours

Speed, Distance, Time

'per' for every
e.g. 80 miles per hour (mph)
Travel 80 miles every hour

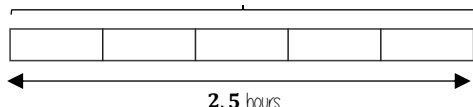
$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

You can use a double number line to help you calculate distance



e.g. A boat travels at a constant speed for 2.5 hours
It travels 300 miles.

300 miles



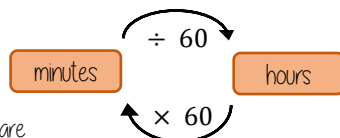
Bar models can help to calculate mph

Each part is half an hour
Each part is 60 miles



Speed, Distance, Time

Before calculations – make sure you are working in the same units as the speed



Learn or learn how to rearrange the formula for speed, distance and time

Substitute in the variables given

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{distance} = \text{speed} \times \text{time}$$

Tables and timetables

Distance tables

This shows the distance between Glasgow and London
It is where their row and column intersects

London		Cardiff		Glasgow		Belfast
211		493				
556		392		177		
518						

Bus/ Train timetables

Harton	1005	1045	1130
Bridge	1024	1106	1147
Avile	1051	1133	1205
Ware	1117	1202	1233

Each column represents a journey, each row represents the time the 'bus' arrives at that location

TIME CALCULATIONS – use a number line

Two-way tables

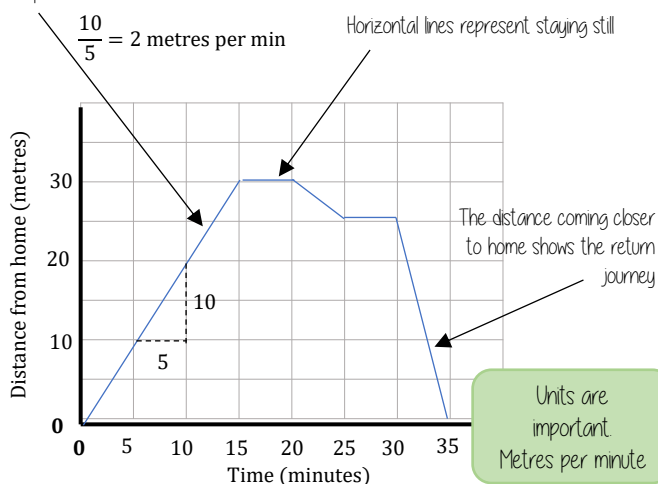
	H	T
H	HH	HT
T	TH	TT

Where rows and columns intersect is the outcome of that action

Distance – Time graphs

The steeper a gradient the faster the speed

Gradient = speed



Units are important
Metres per minute

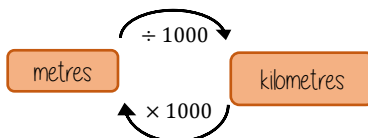
Rates of change & units

Common rates of change relationships

Revisit your conversions between units of length and capacity

Speed: miles per hour

Exchange rates: euros per pounds



NUMBER...

Properties of number

What do I need to be able to do?

- Multiples
- Factors
- Prime numbers
- Write a number as a product of prime factors
- Square, cube and triangular numbers
- Square roots and cube roots
- Explore higher powers and roots (E)
- Highest common factor (HCF)
- Lowest common multiple (LCM)
- HCF and LCM from a Venn diagram (E)
- Use factors to simplify calculations (E)

Keywords

Tier 2:

Multiples: found by multiplying any number by positive integers

Factor: integers that multiply together to get another number.

Prime: an integer with only 2 factors

Conjecture: a statement that might be true (based on reasoning) but is not proven

Counterexample: a special type of example that disproves a statement

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Tier 3:

HCF: highest common factor (biggest factor two or more numbers share)

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Multiples The "times table" of a given number

All the numbers in this lists below are multiples of 3

3, 6, 9, 12, 15...

$3x, 6x, 9x \dots$

This list continues and doesn't end

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

45 is not a multiple of 3 because it is 3×15

Not an integer

Factors

Arrays can help represent factors

5×2 or 2×5

Factors of 10
1, 2, 5, 10

10×1 or 1×10

Factors and expressions

$x \times x \times x \times x \times x \times x$

The number itself is always a factor

Factors of $6x$

$6, x, 1, 6x, 2x, 3, 3x, 2$

$6x \times 1$ OR $6 \times x$

$x \times x$
 $x \times x$

$2x \times 3$

$x \times x \times x$
 $x \times x \times x$

$3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

The first prime number
The only even prime number

2

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

Square and triangular numbers

Square numbers

odd even odd

Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64 ...

Triangular numbers

Representations are useful – an extra counter is added to each new row

Add two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45...

Common factors and HCF

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18 1, 2, 3, 6, 9, 18

30 1, 2, 3, 5, 6, 10, 15, 30

Common factors
(factors of both numbers)
1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

LCM – Lowest common multiple

LCM of 9 and 12

LCM = 36

The first time their multiples match

9 9, 18, 27, 36, 45, 54

12 12, 24, 36, 48, 60

9 12 27 36 45 48

Comparing fractions

$\frac{3}{5}$ and $\frac{7}{10}$

Compare fractions using a LCM denominator

$\frac{6}{10}$ and $\frac{7}{10}$

Higher powers and roots



x^n

n – power
(number of times multiplied by itself)

x – the base number.

Other mental strategies for square roots

$\sqrt{810000}$

$= \sqrt{81} \times \sqrt{10000}$

$= 9 \times 100$

$= 900$

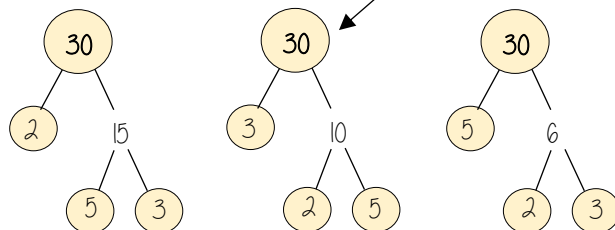


$\sqrt[n]{x}$

Finding the n th root of any value

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

Multiplication is commutative

$30 = 2 \times 3 \times 5$

Multiplication of prime factors

Using prime factors for predictions

eg 60 30×2 $2 \times 3 \times 5 \times 2$
150 30×5 $2 \times 3 \times 5 \times 5$

NUMBER...

Add and subtract fractions

What do I need to be able to do?

- Simplify a fraction
- Convert between mixed numbers and improper fractions
- Add and subtract fractions with the same denominator
- Add and subtract with fractions and integers
- Add and subtract fractions where denominators share a simple common multiple
- Add and subtract fractions with any denominator
- Add and subtract improper fractions and mixed numbers
- Use equivalence to add and subtract decimals and fractions (E)
- Add and subtract simple algebraic fractions (E)
- Substitution and solving equations with fractions (E)

Keywords

Tier 2:

Equivalent: of equal value

Substitute: replace a variable with a numerical value

Tier 3:

Numerator: the number above the line on a fraction. The top number. represents how many parts are taken

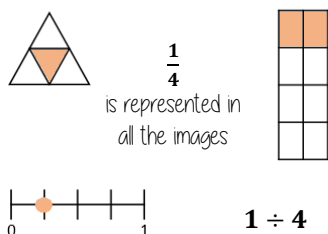
Denominator: the number below the line on a fraction. The number represent the total number of parts

Mixed numbers: a number with an integer and a proper fraction

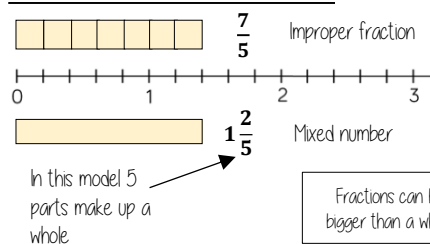
Improper fractions: a fraction with a bigger numerator than denominator

Place value: the value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

Representing Fractions

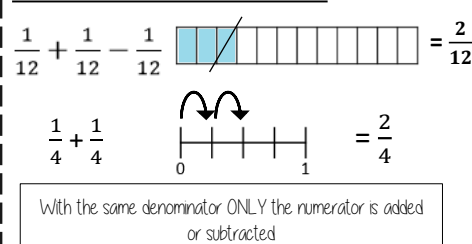


Mixed numbers and fractions



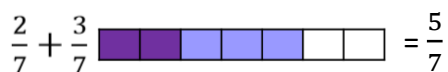
Add/Subtract unit fractions

Same denominator

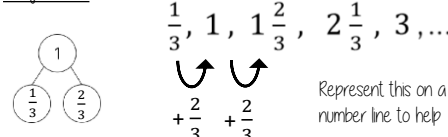


Add/Subtract fractions

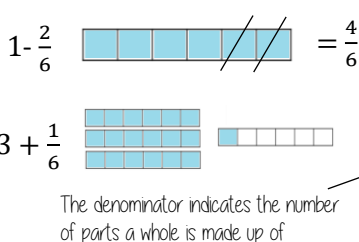
Same denominator



Sequences

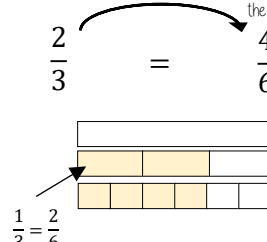


Add/Subtract from integers

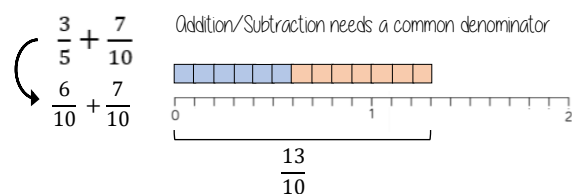


Equivalent fractions

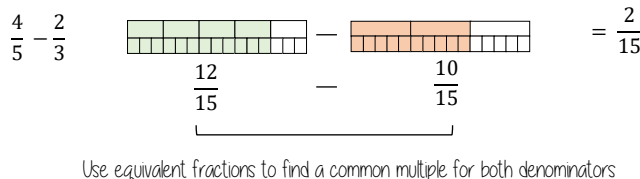
Numerator and denominator have the same multiplier



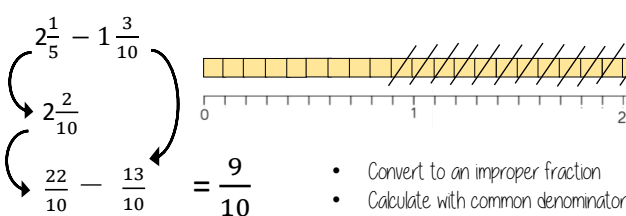
Add/Subtraction fractions (common multiples)



Add/Subtraction any fractions



Add/Subtraction fractions (improper and mixed)



Partitioning method

$$2\frac{1}{5} - 1\frac{3}{10} = 2\frac{2}{10} - 1\frac{3}{10} = 2\frac{2}{10} - 1 - \frac{3}{10} = 1\frac{2}{10} - \frac{3}{10} = \frac{9}{10}$$

Fractions in algebraic contexts

$p = 5$ $m = 2$

$$k - \frac{5}{8} = 2$$

Apply inverse operations

$$k = 2 + \frac{5}{8}$$

$$\frac{7}{9} + \frac{25}{27}$$

Form expressions with fractions

$$b + \frac{7}{9} \rightarrow b + \frac{7}{9}$$

$$\frac{p}{8} + \frac{1}{m}$$

Substitution

$$\frac{5}{8} + \frac{1}{2}$$

Fractions and decimals

$$\frac{1}{10} = 0.1$$

$$\frac{1}{100} = 0.01$$

Example

$$\frac{6}{10} + 0.3$$

$$0.6 + 0.3$$

$$\frac{6}{10} + \frac{3}{10}$$

Remember to use equivalent fractions and common denominators

GEOMETRY...

Angles and polygons

What do I need to be able to do?

- Draw and measure lines and angles
- Understand and use geometric notation
- Angles around a point
- Angles on a straight line
- Vertically opposite angles
- Recognise and name polygons
- Angles in a triangle
- Angles in a quadrilateral
- Solve problems with angles
- Parallel and perpendicular lines
- Angles in parallel lines (E)
- Angles in a polygon (E)
- Simple proofs (E)

Keywords

Tier 2:

Vertically Opposite: angles formed when two or more straight lines cross at a point

Interior Angles: angles inside the shape

Sum: total, add all the interior angles together

Right-angled triangle: a triangle with a right angle

Protractor: equipment used to measure angles

Compass: equipment used to draw arcs and circles

Tier 3:

Convex Quadrilateral: a four-sided polygon where every interior angle is less than 180°

Concave Quadrilateral: a four-sided polygon where one interior angle exceeds 180°

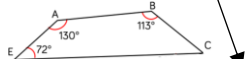
Polygon: A 2D shape made with straight lines

Scalene triangle: a triangle with all different sides and angles

Isosceles triangle: a triangle with two angles the same size and two angles the same size

Letter and labelling convention

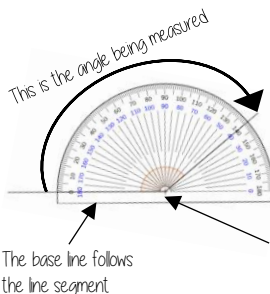
The letter in the middle is the angle
The arc represents the angle



Angle Notation: three letters ABC
This is the angle at B = 113°

Line Notation: two letters EC
The line that joins E to C

Measure angles to 180°

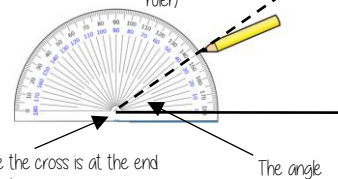


Read from 0°
on the base
line.
Remember to
use estimation.
This is an
obtuse angle so
between 90°
and 180°

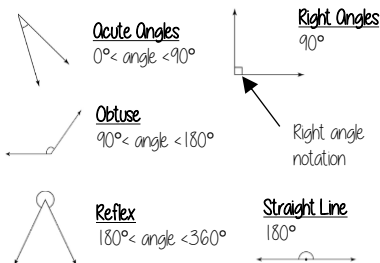
Draw angles up to 180°

Draw a 35° angle

Make a mark at 35° with a pencil
And join to the angle point (use a
ruler)



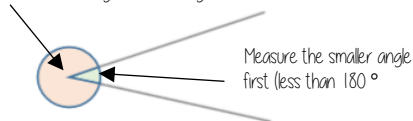
Classify angles



Angles over 180°

Use your knowledge of straight lines 180° and angles around a point 360°

360° - smaller angle = reflex angle



Parallel and Perpendicular lines

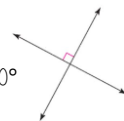
Parallel lines

Straight lines that never meet
(Have the same gradient)



Perpendicular lines

Straight lines that meet at 90°



Properties of Quadrilaterals

Square
All sides equal size
All angles 90°
Opposite sides are parallel



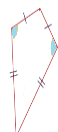
Parallelogram
Opposite sides are parallel
Opposite angles are equal
Co-interior angles

Rectangle
All angles 90°
Opposite sides are parallel



Trapezium
One pair of parallel lines

Rhombus
All sides equal size
Opposite angles are equal



Kite
No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

Polygons

- 3 - Triangle
- 4 - Quadrilateral
- 5 - Pentagon
- 6 - Hexagon
- 7 - Heptagon
- 8 - Octagon
- 9 - Nonagon
- 10 - Decagon



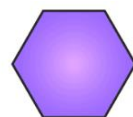
Triangle



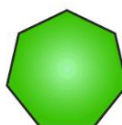
Quadrilateral



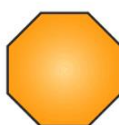
Pentagon



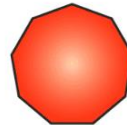
Hexagon



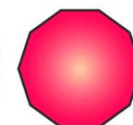
Heptagon



Octagon



Nonagon



Decagon

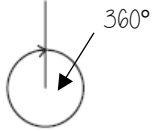
If all the sides and angles are the same, it is a **regular** polygon

GEOMETRY...

Angles and polygons

Sum of angles at a point

The sum of angles around a point is 360°



Find angle BOE

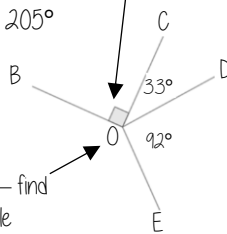
$$90^\circ + 33^\circ + 92^\circ = 205^\circ$$

$$360^\circ - 205^\circ$$

$$\text{BOE} = 155^\circ$$

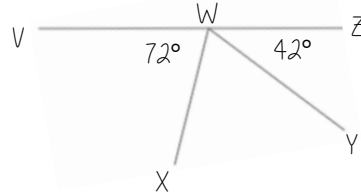
Angle notation - find this missing angle

Angle notation - 90°



Sum of angles on a straight line

Adjacent angles that share a common point on a line add up to 180°

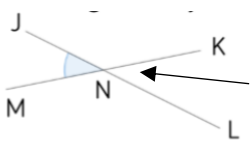


Find angle XWY

$$72^\circ + 42^\circ = 114^\circ$$

$$180^\circ - 114^\circ = 66^\circ$$

Vertically opposite angles

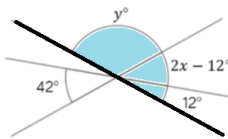


Angle JNM is vertically opposite to angle KNL

$$\text{JNM} = \text{KNL}$$

Vertically opposite angles are the same

Other angle rules still apply
Look for straight line sums and angles around a point.



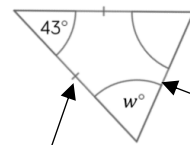
Form equations with information from diagrams:

$$2x - 12 = 42$$

$$2x = 54$$

$$x = 27^\circ$$

Sum of angles in triangles



The two base angles will be the same size

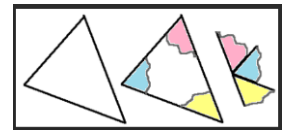
Look at triangle notation
This indicates an isosceles triangle

$$\therefore 180 - 43 = 137$$

$$137 \div 2 = 68.5^\circ$$

A triangle can only have ONE right angle

Sum of interior angles in a triangle = 180°



Have a go!

Tearing the corners from triangles forms a straight line which is therefore 180°

Sum of angles in quadrilaterals

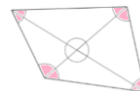
Sum of interior angles in a quadrilateral = 360°



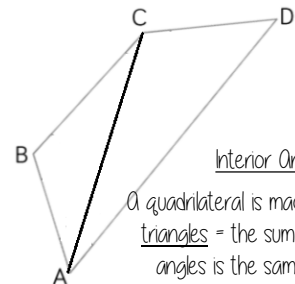
Convex Quadrilateral



Concave Quadrilateral



Interior angles are those that make up the perimeter (outline) of the shape



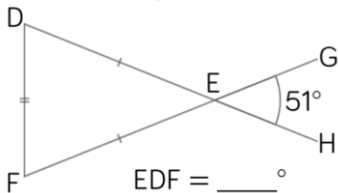
Interior Angles

A quadrilateral is made up of two triangles = the sum of interior angles is the same as two triangles:

$$180^\circ + 180^\circ = 360^\circ$$

Angle Problems

Split up the problem into chunks and explain your reasoning at each point using angle notation



$$\text{EDF} = \underline{\hspace{1cm}}^\circ$$

1. Angle DEF = 51° because it is a vertically opposite angle DEF = GEH

2. Triangle DEF is isosceles (triangle notation) $\therefore \text{EDF} = \text{EFD}$ and the sum of interior angles is 180°
 $180^\circ - 51^\circ = 129^\circ$
 $129^\circ \div 2 = 64.5^\circ$

3. Angle EDF = 64.5°

Keep working out clear and notes together

