YEAR 11
KNOWLEDGE ORGANISERS

White Rose Maths

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PROPORTION...

Ratios and proportion

What do I need to be able to do?

- Solve problems with ratio
- Direct proportion
- Best buy problems
- Conversion graphs
- Exchange rates
- Inverse proportion
- Direct proportion equations (E)
- Inverse proportion equations (E)
- Direct and inverse proportion graphs (E)

Keywords

Tier 2:

Ratio: a statement of how two numbers compare

Equivalent: of equal value

Proportion: a statement that links two ratios

Fraction: represents how many parts of a whole.

Origin: (0,0) on a graph. The point the two axes cross

Gradient: The steepness of a line

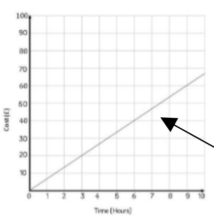
Tier 3:

Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken

Denominator: the number below the line on a fraction. The number represent the total number of parts.

Integer: whole number, can be positive, negative or zero.

Ratio and graphs



Graphs with a constant ratio are directly proportional

- Form a straight line
- Pass through (0,0)

The gradient is the constant ratio

Ratio and scale

A picture of a car is drawn with a scale of 1:30

The car image is 10cm

Image : Real life

$\times 10$ 1cm : 30cm $\times 10$

10cm : 300cm

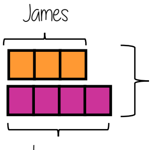
Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4
Work out how much each person earns

Model the Question

James: Lucy

3 : 4



Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

£350 ÷ 7 = £50

☐ = one part = £50

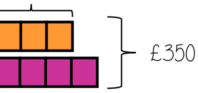
Put back into the question

James: Lucy

x 50 3 : 4 x 50

£150:£200

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

Ratios in 1 : n and n : 1

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1 : n

The question states that this part has to be 1 unit Therefore Divide by 4

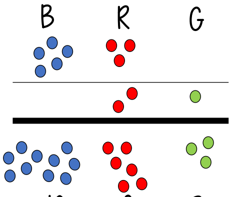
4 : 20

1 : 5

This side has to be divided by 4 too – to keep in proportion

the n part does not have to be an integer for this type of question

Combining ratios



The ratio of Blue to Red is 5:3

The ratio of Red to Green is 2:1

Ratio of Blue to Red to Green

10 : 6 : 3

Use equivalent ratios to allow comparison of the group that is common to both statements

Lowest common multiple of the ratio both statements share

Conversion between currencies

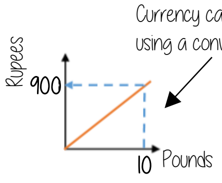
£1 = 90 Rupees ← Currency is directly proportional

For every £1 I have 90 Rupees

$\times 10$ £1 = 90 Rupees $\times 10$

£10 = 900 Rupees

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

$\times 7$ £1 = 90 Rupees $\times 7$ ← 630 ÷ 90 = 7

£7 = 630 Rupees

Best buys

4 pens costs £2.60

10 pens costs £6.00

You could work out how much 40 pens are and then compare

"1 pen costs..." £2.60 ÷ 4 = £0.65 £6.00 ÷ 10 = £0.60

"1-pound buys..." 4 ÷ 2.60 = 1.54 pens 10 ÷ 6 = 1.67 pens

Compare the solution in the context of the question

The best value has the lowest cost "per pen"

The best value means £1 buys you more pens

Direct and Inverse Proportion equations (H)

When y is directly proportional to x:
y=kx for a constant k

$\boldsymbol{b}$ is directly proportional to $\boldsymbol{a^2}$;

a = 6 when $\boldsymbol{b} = 90$. Find $\boldsymbol{b}$ if a = 8.

To find $\boldsymbol{k}$: $\boldsymbol{b}=\boldsymbol{k}\boldsymbol{a}^2$; $\boldsymbol{a}=6$ and $\boldsymbol{b}=90$

$\mathbf{90 = \boldsymbol{k}\times 6^2}$ **so** $\boldsymbol{k} = \mathbf{2.5,}$

Thus $\boldsymbol{b} = \mathbf{2.5}\boldsymbol{a}^2$ and when a = 8

$\boldsymbol{b} = \mathbf{2.5\times 8^2 = 160}$

When y is inversely proportional to x:
$yx = k$ or $y = \frac{k}{x}$ for a constant k

GEOMETRY…

Area and Volume

What do I need to be able to do?

- Isometric drawings
- Plans and elevations
- Volume of a cylinder
- Volume of a sphere
- Volume of cones and pyramids
- Surface area of a sphere
- Surface area of a cylinder (E)
- Surface area of pyramids (E)
- Surface area of cones (E)
- Convert metric units of area (E)
- Convert metric units of volume (E)
- Volume and surface area of a frustum (E)

Keywords

Tier 2:

Area: the size of the 2D surface

Diameter: the distance from one side of a circle to another through the centre

Radius: the distance from the centre to the circumference of the circle

Tangent: a straight line that touches the circumference of a circle

Chord: a line segment connecting two points on the curve

Hemisphere: half a sphere

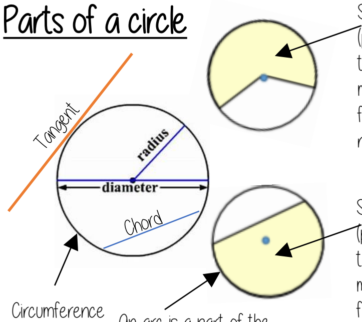
Surface area: the total area of the surface of a 3D shape.

Tier 3:

Circumference: the length around the outside of the circle – the perimeter

Frustum: a pyramid or cone with the top cut off

Parts of a circle



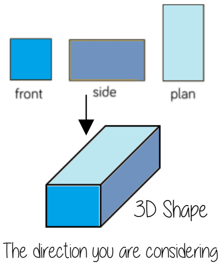
Sector (part of the circle made from two radii)

Segment (part of the circle made from a chord)

Circumference

On arc is a part of the circumference

Plans and elevations

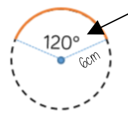


The direction you are considering the shape from determines the front and side views

Sector area

Remember a sector is part of a circle

Area of the whole circle = πr^2 = $\pi \times 6^2 = 36\pi$



Sector area = $\frac{\theta}{360} \times$ area of circle

$= \frac{120}{360} \times 36\pi$

$= \frac{1}{3} \times 36\pi = \underline{12\pi}$

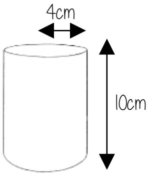
Volume of a cone and a cylinder

Volume Cylinder= $\pi r^2 h$

A cylinder is a prism – cross section is a circle

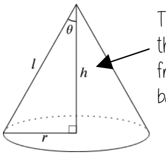
Volume Cone = $\frac{1}{3}\pi r^2 h$

A cone is a pyramid with a circular base



$$V = \pi r^2 h$$
$$= \pi \times 42 \times 10$$
$$= \pi \times 160$$
$$= \underline{160\pi}\ \text{cm}^2$$

Give your answer in terms of π' means NOT in terms of pi $= \underline{502.7}$cm^2



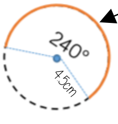
The height of a cone is the perpendicular height from the vertex to the base

Look out for trigonometry or Pythagoras theorem – the radius forms the base of a right-angled triangle

Arc length

Remember an arc is part of the circumference

Circumference of the whole circle = πd = $\pi \times 9 = 9\pi$



Arc length = $\frac{\theta}{360} \times$circumference

$= \frac{240}{360} \times 9\pi$

$= \frac{2}{3} \times 9\pi$ $= \underline{6\pi}$

Perimeter

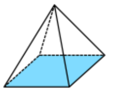
Perimeter is the length around the outside of the shape

This includes the arc length and the radii that encloses the shape

Perimeter = $\frac{\theta}{360} \times$circumference $+ 2r$ $= \underline{6\pi + 9}$

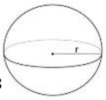
Pyramid

Volume Pyramid = $\frac{1}{3}$ x base area x h



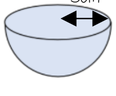
The surface area of a pyramid can be calculated by adding the area of the base to the sum of the areas of the triangular faces

Volume of a sphere



Volume Sphere = $\frac{4}{3}\pi r^3$

NOTE: This is now a **cubed** value



Volume Sphere = $\frac{4}{3}\pi r^3$

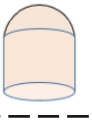
$= \frac{4}{3} \times \pi \times 3^3$

$= \frac{4}{3} \times \pi \times 27$ $= \underline{36\pi}$

A hemisphere is half the volume of the overall sphere $= 36\pi \div 2$

$= 18\pi$

Look out for hemispheres being placed on other 3D shapes, e.g cones and cylinders

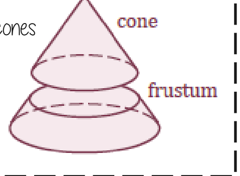


Frustum

A frustum is the base part of a solid cone or pyramid formed by cutting off the top by a plane parallel to the base

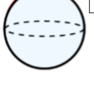
Volume of frustum is difference between the volumes of two cones

The surface area of a frustum (like a lampshade or cut cone/pyramid) involves finding the **curved surface area** and adding the areas of the **top and bottom faces**



Surface area of a sphere

Surface area = $4\pi r^2$



Radius = 5cm

Surface area = $4\pi r^2$

$= 4 \times \pi \times 5^2$

$= 4 \times \pi \times 25$

The curved surface area of a sphere $= \underline{100\pi}$

A hemisphere has the curved surface AND a flat circular face

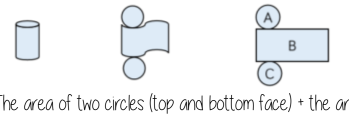
$= 100\pi \div 2 = 50\pi$

$= 50\pi + \pi \times 5^2$

Hemisphere $= \underline{75\pi}$

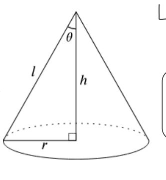
Surface area of cones and cylinders

Surface area cylinder= $2\pi r^2 + \pi dh$



The area of two circles (top and bottom face) + the area of the curved face

The length of shape B is the circumference of the circles



Curved surface area Cone = $\pi r l$

Look out for the use of Pythagoras to calculate the length l

Total surface area = curved face + circle face (area of base)

GEOMETRY… Similarity and congruence

What do I need to be able to do?

- Find unknown sides and angles in similar shapes
- Identify similar triangles
- Solve problems with similar shapes
- Areas of similar shapes
- Volumes of similar shapes
- Solve problems with area and volume in similar shapes
- Similarity and congruence
- Congruent triangles
- Prove a pair of triangles are congruent (E)

Keywords

Tier 2:

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)

Similar: when one shape can become another with a reflection, rotation, enlargement or translation.

Congruent: the same size and shape

Corresponding: items that appear in the same place in two similar situations

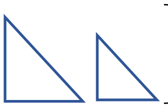
Parallel: straight lines that never meet (equal gradients)

Tier 3:

Scale Factor: the multiplier of enlargement

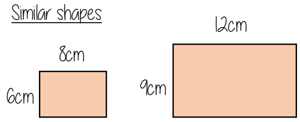
Centre of enlargement: the point the shape is enlarged from

Identify similar shapes



Angles in similar shapes do not change
e.g. if a triangle gets bigger the angles can not go above 180°

Similar shapes

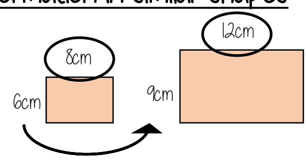


Scale Factor:
Both sides on the bigger shape are 1.5 times bigger

Compare sides: 6 : 9 → 2 : 3 8 : 12 → 2 : 3

Both sets of sides are in the same ratio

Information in similar shapes



Compare the equivalent side on both shapes

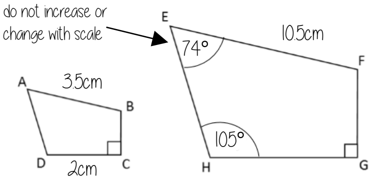
Scale Factor is the multiplicative relationship between the two lengths

Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

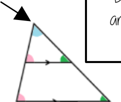
AB and EF are corresponding

Remember angles do not increase or change with scale



Similar triangles

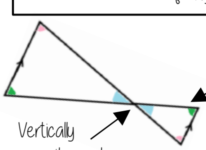
Shares a vertex



Because corresponding angles are equal the highlighted angles are the same size

Parallel lines – all angles will be the same in both triangle

As all angles are the same this is similar – it only one pair of sides are needed to show equality

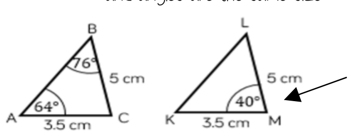


All the angles in both triangles are the same and so similar

Vertically opposite angles

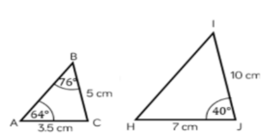
Congruence and Similarity

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**



Because all angles are the same, but all sides are enlarged by 2 ABC and HIJ are **similar**

Conditions for congruent triangles

Triangles are congruent if they satisfy any of the following conditions

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

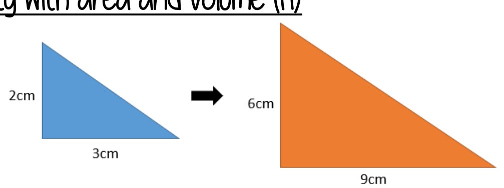
Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

Similarity with area and volume (H)

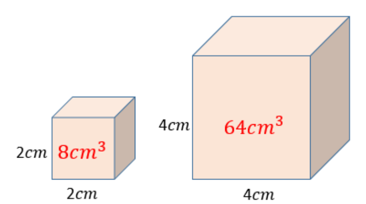


Area = 3cm² Area = 27cm²

	Shape A		Shape B
Length	2cm	x3 →	6cm
Area	3cm²	x3² →	27cm²

If the length increases by a scale factor k, the **area** increases by this squared, k^2

If the length increases by a scale factor k, the **volume** increases by this cubed, k^3



	Shape A		Shape B
Length	2cm	x2 →	4cm
Area	4cm²	x2² →	16cm²
Volume	8cm³	x2³ →	64cm³

ALGEBRA...

Sequences and proof

What do I need to be able to do?

- Describe and continue sequences
- Explore sequences with algebraic terms
- Describe and continue sequences involving surds (E)
- Generate a sequence given an algebraic rule
- nth term of a linear sequence
- nth term of a quadratic sequence (E)
- Represent numbers algebraically
- Algebraic arguments and proof

Keywords

Tier 2:
Factor: numbers we multiply together to make another number
Multiple: the result of multiplying a number by an integer.
Arithmetic: a sequence where the difference between the terms is constant
Sequence: items or numbers put in a pre-decided order
Tier 3:
Fibonacci: a sequence where each number is the sum of the two that precede it
Quadratic: a number sequence with a constant second difference
Geometric: a sequence where each term is found by multiplying the previous one by a fixed nonzero number

Linear and Non Linear Sequences

Linear Sequences – increase by addition or subtraction and the same amount each time

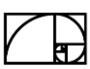
Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division.

Fibonacci Sequence – look out for this type of sequence

0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms.



Sequences from algebraic rules

This is substitution!

$3n + 7$

This will be linear - note the single power of n. The values increase at a constant rate

$3n^2 + 7$

This is not linear as there is a power for n

$2n - 5$ →

Substitute the number of the term you are looking for in place of 'n'

e.g.
1st term = 2 (1) – 5 = -3
2nd term = 2 (2) – 5 = -1
100th term = 2 (100) – 5 = 195

Checking for a term in a sequence

Form an equation

Is 201 in the sequence 3n – 4?

$3n - 4 = 201$

Algebraic rule

Term to check

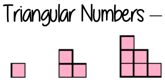
Solving this will find the position of the term in the sequence.
ONLY an integer solution can be in the sequence.

Other sequences

Fibonacci Sequence
1, 1, 2, 3, 5, 8 ...

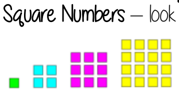
Each term is the sum of the previous two terms

Triangular Numbers – look at the formation



1, 3, 6, 10, 15

Square Numbers – look at the formation



1, 4, 9, 16...

Sequences are the repetition of a patten

Arithmetic/ Geometric sequences

Arithmetic Sequences change by a common difference. This is found by addition or subtraction between terms

Geometric Sequences change by a common ratio. This is found my multiplication/ division between terms.

Term to term rule – how you get from one term (number in the sequence) to the next term

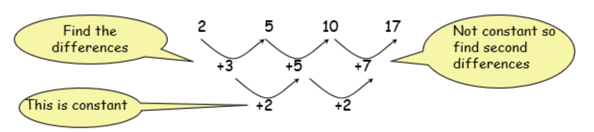
Position to term rule – take the rule and substitute in a position to find a term. E.g. Multiply the position number by 3 and then add 2

Finding the nth term of a quadratic sequence(H)

1. Find the first and second differences.
2. Halve the second difference and multiply this by n^2.
3. Substitute n= 1,2,3,4... into your expression so far.
4. Subtract this set of numbers from the corresponding terms in the sequence from the question.
5. Find the nth term of this set of numbers.
6. Combine the nth terms to find the overall nth term of the quadratic sequence.

Example: Find the formula for the nth term of the sequence:

2, 5, 10, 17



The second difference is **CONSTANT** so the formula for the nth term must contain a n^2. The number in front of n^2 is **half** the constant difference.

n^2

Term number	1	2	3	4
Sequence	2	5	10	17
	+ 1	+1	+1	+1
n^2	1	4	9	16

This is constant so now we can work out the nth term

$n^2 + 1$

Finding the nth term of an arithmetic sequence

This is the 4 times table $4n$ → 4, 8, 12, 16, 20...

7, 11, 15, 19, 22

This has the same constant difference – but is 3 more than the original sequence

$4n + 3$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

Complex algebraic rules

Misconceptions and comparisons

$2n^2$

2 times whatever n squared is

$(2n)^2$

2 times n then square the answer

NUMBER...

Standard Form

What do I need to be able to do?

- Numbers greater than 1 in standard form
- Numbers between 0 and 1 in standard form
- Adjust a number to standard form
- Multiply and divide numbers in standard form (E)
- Add and subtract numbers in standard form (E)
- Four operations with numbers in standard form
- Solve problems with standard form

Keywords

Tier 2:
Standard (index) Form: A system of writing very big or very small numbers
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Negative: A value below zero.
Tier 3:
Commutative: an operation is commutative if changing the order does not change the result.

Positive powers of 10

1 billion – 1 000 000 000
$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

Addition rule for indices $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices $10^a \div 10^b = 10^{a-b}$

Numbers between 0 and 1

0.054
$= 5.4 \times 10^{-2}$

1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^0	10^{-1}	10^{-2}	10^{-3}
0	0	5	4

A negative power does not mean a negative answer – it means a number closer to 0

Standard form with numbers > 1

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Example
3.2×10^4
$= 3.2 \times 10 \times 10 \times 10 \times 10$
$= 32000$

Non-example
0.8×10^4
$5.3 \times 10^{0.7}$

Negative powers of 10

0.001
$1 \times \frac{1}{1000}$
1×10^{-3}

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
0	0	0	0	1

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Order numbers in standard form

10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}

6.4×10^{-2}	2.4×10^2	3.3×10^0	1.3×10^{-1}
0.064	240	1	0.13

Look at the power first will the number be = > or < than 1

Use a place value grid to compare the numbers for ordering

Mental calculations

$6.4 \times 10^2 \times 1000$ Not in Standard Form
$= 6.4 \times 10^2 \times 10^3$ Use addition for indices rule
$= 6.4 \times 10^5$

$8 \times 10^5 \times 3$
$= 24 \times 10^5$ Not in Standard Form
$= 2.4 \times 10^1 \times 10^5$ Use addition for indices rule
$= 2.4 \times 10^6$

$(2 \times 10^3) \div 4$ Divide the values
$= (2 \div 4) \times 10^3$
$= 0.5 \times 10^3$

Remember the layout for standard form

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$6 \times 10^5 + 8 \times 10^5$

Method 1
$= 600000 + 800000$
$= 1400000$
$= 1.4 \times 10^5$

More robust method
Less room for misconceptions
Easier to do calculations with negative indices
Can use for different powers

Method 2
$= (6 + 8) \times 10^5$
$= 14 \times 10^5$ ← This is not the final answer
$= 1.4 \times 10^1 \times 10^5$
$= 1.4 \times 10^5$

Only works if the powers are the same

Multiplication and division

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$ ← Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \times 10^5 \div 10^3$

$= 5 \times 10^2$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

Revisit addition and subtraction laws for indices – they are needed for the calculations

Addition law for indices
$a^m \times a^n = a^{m+n}$

Subtraction law for indices
$a^m \div a^n = a^{m-n}$

Using a calculator

$14 \times 10^5 \times 3.9 \times 10^3$

Use a calculator to work out this question to a suitable degree of accuracy

Input 14 and press $\boxed{\times 10^x}$ Then press 5 (for the power)
Press $\boxed{\times}$
Input 3.9 and press $\boxed{\times 10^x}$ Then press 3 (for the power)
Press $\boxed{=}$

This gives you the solution

Click calculator for video tutorial

To put into standard form and a suitable degree of accuracy
Press $\boxed{\text{SHIFT}}$ $\boxed{\text{SETUP}}$ and then press 7 for sci mode
Choose a degree of accuracy so in most cases press 2

Answer: 5.5×10^8

GEOMETRY...

Work with circles(F)

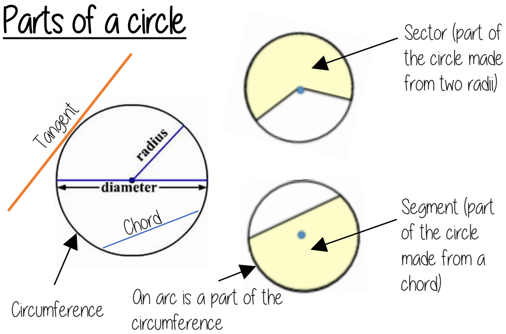
What do I need to be able to do?

- Area and circumference of a circle
- Fractional parts of a circle (area)
- Area of a sector (E)
- Fractional parts of a circle (perimeter)
- Arc length and perimeter (E)
- Area and perimeter of compound shapes with circles
- Solve problems with circles

Keywords

Tier 2:
Area: the size of the 2D surface
Diameter: the distance from one side of a circle to another through the centre
Radius: the distance from the centre to the circumference of the circle
Tangent: a straight line that touches the circumference of a circle
Chord: a line segment connecting two points on the curve
Hemisphere: half a sphere
Surface area: the total area of the surface of a 3D shape.
Tier 3:
Circumference: the length around the outside of the circle – the perimeter

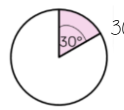
Parts of a circle



Fractional parts of a circle

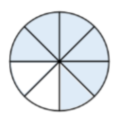
A circle is made up of 360°

Formula to remember:
Area of a circle = πr^2
Circumference of a circle = πd or $2\pi r$



30° represents $\frac{30}{360}$ of a full circle

$\frac{30}{360} = \frac{1}{12}$



$\frac{270}{360}$ of a full circle (in degrees)

$\frac{6}{8}$ of a full circle (in equal parts)

$\frac{3}{4}$ of a full circle

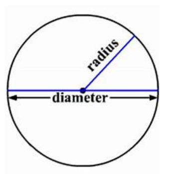
The fraction of the circle is as $\frac{\theta}{360}$

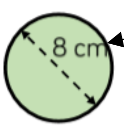
θ represents the degrees in the sector

Area of a circle (Non-Calculator)

Read the question – leave in terms of π or if $\pi \approx 3$ (provides an estimate for answers)

Area of a circle
π x radius^2





Diameter = 8cm
∴ Radius = 4cm

π x radius^2
= π x 4^2
= π x 16
= 16π cm^2

Find the area of one quarter of the circle

Radius = 4cm

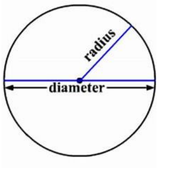
Circle Area = 16π cm^2
Quarter= 4π cm^2

Area of a circle (Calculator)

SHIFT x10^x

How to get π symbol on the calculator

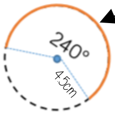
Area of a circle
π x radius^2



It is important to round your answer suitably – to significant figures or decimal places. This will give you a decimal solution that will go on forever!

Arc length

Remember an arc is part of the circumference
Circumference of the whole circle = πd = $\pi \times 9 = 9\pi$



Arc length = $\frac{\theta}{360}$ ×circumference

$= \frac{240}{360} \times 9\pi$

$= \frac{2}{3} \times 9\pi$ = $\underline{6\pi}$

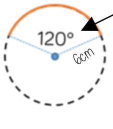
Perimeter
Perimeter is the length around the outside of the shape
This includes the arc length and the radii that encloses the shape

Perimeter = $\frac{\theta}{360}$ ×circumference $+ 2r$

= $\underline{6\pi + 9}$

Sector area

Remember a sector is part of a circle
Area of the whole circle = $\pi r^2 = \pi \times 6^2 = 36\pi$



Sector area = $\frac{\theta}{360}$ × area of circle

$= \frac{120}{360} \times 36\pi$

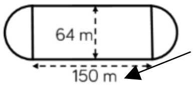
$= \frac{1}{3} \times 36\pi = \underline{12\pi}$

Compound shapes including circles

Circumference
π x diameter

Compound shapes are not always area questions. For Perimeter you will need to use the circumference

Spotting diameters and radii



This dimension is also the diameter of the semi circles.

Arc lengths = π x 64
= 64 π

Don't need to halve this because there are 2 ends which make the whole circle

Arc lengths + Straight lengths = total perimeter

= 64 π + 150 + 150
= $\underline{(300 + 64\pi)}$ m
OR = $\underline{501 \text{ m}}$

Still remember to split up the compound shape into smaller more manageable individual shapes first

GEOMETRY… Circle Theorems(H)

What do I need to be able to do?

- Angles in circles
- Circle theorem (angles at the centre and circumference)
- Circle theorem (angles in a semicircle)
- Circle theorem (angles in the same segment)
- Circle theorem (angles in a cyclic quadrilateral)
- Circle theorem (angle between radius and chord)
- Circle theorem (angle between radius and tangent)
- Circle theorem (two tangents from a point)
- Circle theorem (alternate segment theorem)
- Solve problems with circle theorems

Keywords

Tier 2:

Area: the size of the 2D surface

Diameter: the distance from one side of a circle to another through the centre

Radius: the distance from the centre to the circumference of the circle

Tangent: a straight line that touches the circumference of a circle

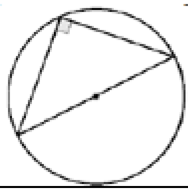
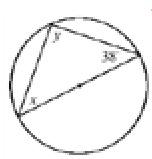
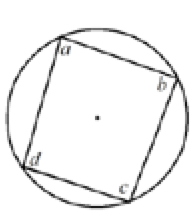
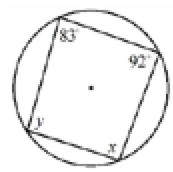
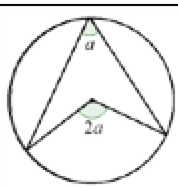
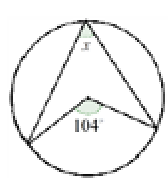
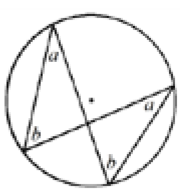
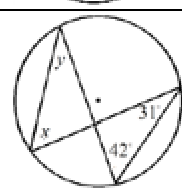
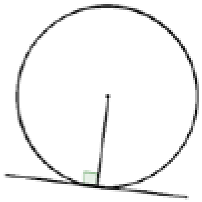
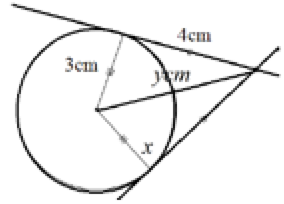
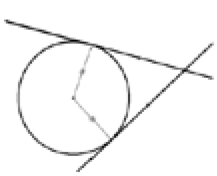
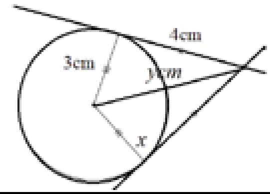
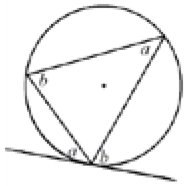
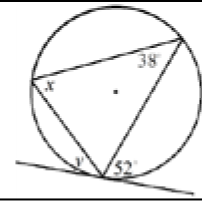
Chord: a line segment connecting two points on the curve

Hemisphere: half a sphere

Surface area: the total area of the surface of a 3D shape.

Tier 3:

Circumference: the length around the outside of the circle – the perimeter

	Angles in a semi-circle have a right angle at the circumference.		$y = 90°$ $x = 180 - 90 - 38 = 52°$
 $a+c=180°$ $b+d=180°$	Opposite angles in a cyclic quadrilateral add up to 180°		$x = 180 - 83 = 97°$ $y = 180 - 92 = 88°$
	The angle at the centre is twice the angle at the circumference.		$x = 104 \div 2 = 52°$
	Angles in the same segment are equal		$x = 42°$ $y = 31°$
	A tangent is perpendicular to the radius at the point of contact		$y = 5cm$ (Pythagoras' Theorem)
	Tangents from an external point that are equal in length		$x = 90°$
	Alternate Segment Theorem		$x = 52°$ $y = 38°$

PROBABILITY… Set Notation and Venn Diagrams

What do I need to be able to do?

- Set notation
- Venn diagrams
- Probability from Venn diagrams
- Solve problems with Venn diagrams
- Conditional probability (Venn diagrams and two-way tables) (E)

Keywords

Tier 2:

Outcomes: the result of an event that depends on probability.

Probability: the chance that something will happen.

Set: a collection of objects.

Chance: the likelihood of a particular outcome.

Event: the outcome of a probability – a set of possible outcomes.

Biased: a built-in error that makes all values wrong by a certain amount.

Union: Notation '∪' meaning the set made by comparing the elements of two sets.

Identify and represent sets

The **universal set** has this symbol ξ – this means EVERYTHING in the Venn diagram is in this set

A set is a collection of things – you write sets inside curly brackets { }

ξ = {the numbers between 1 and 50 inclusive}

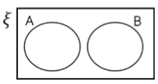
My sets can include every number between 1 and 50 including those numbers

A = {Square numbers}

A = {1, 4, 9, 16, 25, 36, 49}

All the numbers in set A are square number and between 1 and 50

Interpret and create Venn diagrams



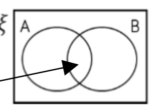
Mutually exclusive sets

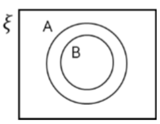
The two sets have nothing in common

No overlap

Union of sets

The two sets have some elements in common – they are placed in the intersection





Subset

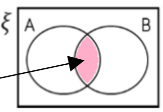
All of set B is also in Set A so the ellipse fits inside the set

The box

Around the outside of every Venn diagram will be a box. If an element is not part of any set it is placed outside an ellipse but inside the box

Intersection of sets

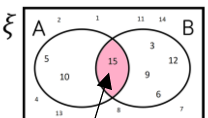
Elements in the intersection are in set A AND set B



The notation for this is $A \cap B$

ξ = {the numbers between 1 and 15 inclusive}

A = {Multiples of 5} B = {Multiples of 3}

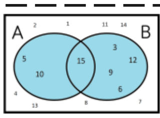


The element in $A \cap B$ is 15

In this example there is only one number that is both a multiple of 3 and a multiple of 5 between 1 and 15

Union of sets

Elements in the union could be in set A OR set B



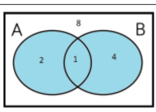
The notation for this is $A \cup B$

ξ = {the numbers between 1 and 15 inclusive}

A = {Multiples of 5} B = {Multiples of 3}

The elements in $A \cup B$ are 5, 10, 15, 3, 9, 6, 12

There are 7 elements that are either a multiple of 5 OR a multiple of 3 between 1 and 15



This Venn shows the **number of elements** in each set

Sample space – for single events

A sample space for rolling a six-sided dice is S = {1, 2, 3, 4, 5, 6}

A sample space for this spinner is S = {Pink, Blue, Yellow}

You only need to write each element once in a sample space diagram

- A Sample space represents a possible outcome from an event
- They can be interpreted in a variety of ways because they do not tell you the probability

Probability of a single event

Probability = $\frac{\text{number of times event happens}}{\text{total number of possible outcomes}}$

Probability notation P (event)

P(Blue) = $\frac{4}{10}$ ← There are 4 blue sectors; There are 10 sectors overall

$= \frac{2}{5}$

Probability can be a fraction, decimal or percentage value

$\frac{4}{10} = \frac{40}{100} = 0.40 = 40\%$

Probability is always a value between 0 and 1

Sum of probabilities

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$

∴The probability of **NOT** getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

P(white chocolate) = 1 – 0.15 – 0.35

= 0.5

Independent events

The rolling of one dice has no impact on the rolling of the other. The individual probabilities should be calculated separately

Probability of event 1 × Probability of event 2

$P(5) = \frac{1}{6}$ $P(R) = \frac{1}{4}$

Find the probability of getting a 5 and a red

$P(5\ and\ R) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

PROBABILITY… Set Notation and Venn Diagrams

Construct sample space diagrams

Sample space diagrams provide a systematic way to display outcomes from events

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

This is the set notation to list the outcomes S =

In between the { } are a;; the possible outcomes

S = { 1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T}

Probability from sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

What is the probability that an outcome has an even number and a tails?

This is the set notation that represents the question P

P (Even number and Tails) = $\frac{3}{12}$

There are three even numbers with tails

There are twelve possible outcomes

In between the ( ) is the event asked for

Numerator: the event

Denominator: the total number of outcomes

Probability from two-way tables

	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

The total number of items

P (Girl walk to school) = $\frac{21}{100}$

The event

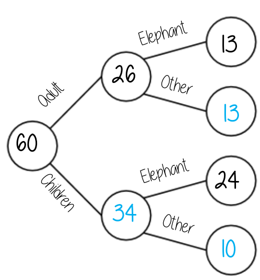
The total in the set

Product Rule

The number of items in event a X The number of items in event b

Probability from frequency trees

60 people visited the zoo one Saturday morning.
26 of them were adults.
13 of the adult's favourite animal was an elephant.
24 of the children's favourite animal was an elephant.



Frequency trees and two-way tables can show the same information

The total columns on two-way tables show the possible denominators

P(adult) = $\frac{26}{60}$

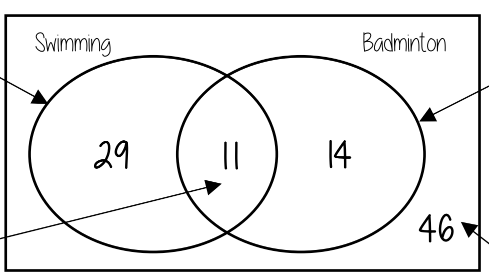
P(Child with favourite animal as elephant) = $\frac{13}{37}$

Probability from Venn diagrams

100 students were questioned if they played badminton or went to swimming club.
40 went swimming, 25 went to badminton and 11 went to both.

This whole curve includes everyone that went swimming. Because 11 did both we calculate **just** swimming by 40- 11

The intersection represents both Swimming **AND** badminton



This whole curve includes everyone that went to badminton. Because 11 did both we calculate **just** badminton by 25 - 11

The number outside represents those that did **neither** badminton or swimming

100 – 29 – 11 – 14

P (Just swimming) = $\frac{29}{100}$

GEOMETRY...

Vectors(H)

What do I need to be able to do?

- Solve problems with vectors
- Ratios with vectors (E)
- Collinear points using vectors (E)
- Geometric arguments and proofs with vectors (E)

Keywords

Tier 2:
Direction: the line our course something is going
Magnitude: the magnitude of a vector is its length
Resultant: the vector that is the sum of two or more other vectors
Parallel: straight lines that never meet
Tier 3:
Column vector: a matrix of one column describing the movement from a point
Scalar: a single number used to represent the multiplier when working with vectors

Understand and represent vectors

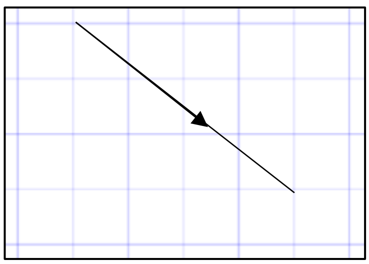
Vectors show both direction and magnitude

Column vectors have been seen in translations to describe the movement of one image onto another

Movement along the x-axis

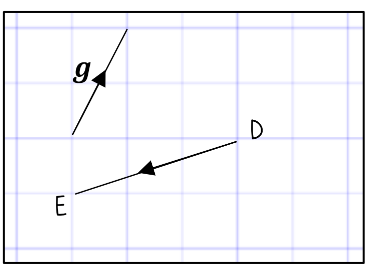
Movement along the y-axis

$\begin{pmatrix}4\\-3\end{pmatrix}$



The arrow is pointing in the direction from starting point to end point of the vector.	The direction is important to correctly write the vector
The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)	The magnitude stays the same even if the direction changes

Understand and represent vectors



Vector notation $\overrightarrow{DE}$ is another way to represent the vector joining the point D to the point E.

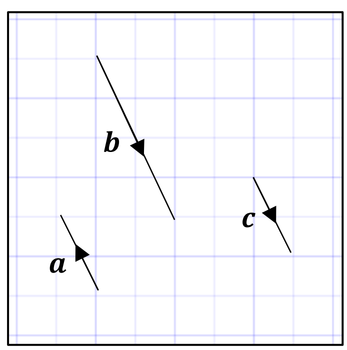
$\overrightarrow{DE} = \begin{pmatrix}-3\\-1\end{pmatrix}$

The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so $\boldsymbol{g}$ represents the vector $\boldsymbol{g} = \begin{pmatrix}1\\2\end{pmatrix}$

Vectors multiplied by a scalar

Parallel vectors are scalar multiples of each other



$\boldsymbol{b} = 2 \times \boldsymbol{c} = 2\boldsymbol{c}$

Multiply $\boldsymbol{c}$ by 2 this becomes $\boldsymbol{b}$. The two lines are parallel

$\boldsymbol{a} = -1 \times \boldsymbol{c} = -\boldsymbol{c}$

The vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are also parallel. A negative scalar causes the vector to reverse direction.

$\boldsymbol{b} = -2 \times \boldsymbol{a} = -2\boldsymbol{a}$

$\boldsymbol{a} = \begin{pmatrix}-1\\2\end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix}2\\-4\end{pmatrix}$ $\boldsymbol{c} = \begin{pmatrix}1\\-2\end{pmatrix}$

Addition of vectors

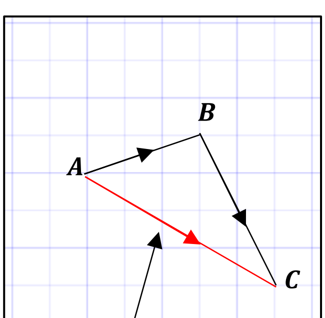
$\overrightarrow{AB} = \begin{pmatrix}3\\1\end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix}2\\-4\end{pmatrix}$

$$\overrightarrow{AB} + \overrightarrow{BC}$$

$$= \begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}2\\-4\end{pmatrix}$$

$$= \begin{pmatrix}3+2\\1+-4\end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}$$

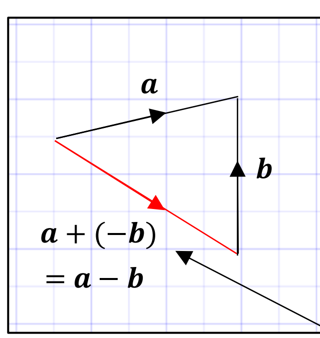


The resultant

Look how this addition compares to the vector $\overrightarrow{AC}$

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}$$

Addition and subtraction of vectors



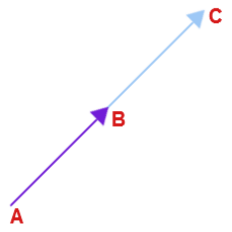
$\boldsymbol{a} = \begin{pmatrix}5\\1\end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix}0\\4\end{pmatrix}$

$$\boldsymbol{a} + (-\boldsymbol{b}) = \begin{pmatrix}5+ & -0\\1+ & -4\end{pmatrix} = \begin{pmatrix}5\\-4\end{pmatrix}$$

The resultant is $\boldsymbol{a} - \boldsymbol{b}$ because the vector is in the opposite direction to b which needs a scalar of -1

Collinear Vectors(H)

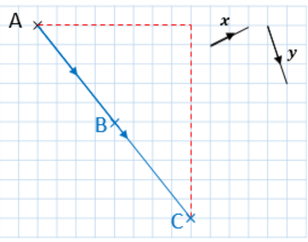
Collinear vectors are vectors that are on the same line.



To show that two vectors are collinear, show that one vector is a multiple of the other (parallel)

AND

that both vectors share a point.



$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel

AB and BC both pass through the point B

A, B and C are collinear.

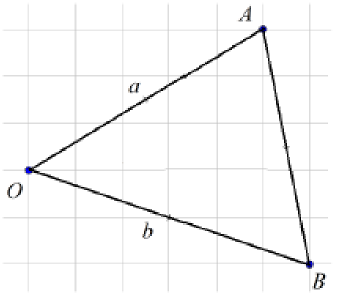
$\overrightarrow{AB} = \boldsymbol{x} + 2\boldsymbol{y}$

$= \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$\overrightarrow{AC} = 2\boldsymbol{x} + 4\boldsymbol{y}$

$= \begin{pmatrix} 8 \\ -10 \end{pmatrix}$

Vector Geometry(H)

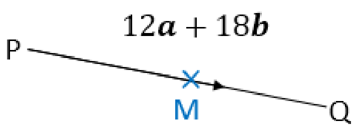


$\overrightarrow{OA} = a \qquad \overrightarrow{AO} = -a$

$\overrightarrow{OB} = b \qquad \overrightarrow{BO} = -b$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -a + b = b - a$

$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -b + a = a - b$

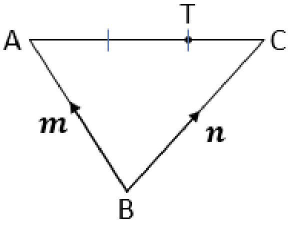


M is the midpoint of PQ.

Find the vector $\overrightarrow{PM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

$\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ}$

$\overrightarrow{PM} = \frac{1}{2}(12\boldsymbol{a} + 18\boldsymbol{b}) \quad = 6\boldsymbol{a} + 9\boldsymbol{b}$



T lies on AC such that AT : TC = 2 : 1

Find $\overrightarrow{BT}$

$\overrightarrow{AC} = \boldsymbol{n} - \boldsymbol{m}$

$\overrightarrow{BT} = \overrightarrow{BA} + \frac{2}{3}\overrightarrow{AC}$

$\overrightarrow{BT} = \boldsymbol{m} + \frac{2}{3}(\boldsymbol{n} - \boldsymbol{m})$

$\overrightarrow{BT} = \boldsymbol{m} + \frac{2}{3}\boldsymbol{n} - \frac{2}{3}\boldsymbol{m}$

$\overrightarrow{BT} = \frac{1}{3}\boldsymbol{m} + \frac{2}{3}\boldsymbol{n}$

$\overrightarrow{BT} = \frac{1}{3}(\boldsymbol{m} + 2\boldsymbol{n})$

ALGEBRA... Functions and graphs

What do I need to be able to do?

- Function machines
- Write an expression or formula from a function machine
- Quadratic graphs
- Cubic graphs
- Reciprocal graphs
- Recognise and sketch graphs

Keywords

Tier 2:

Inverse: the operation that undoes what was done by the previous operation. (The opposite operation)
Function: a relationship that instructs how to get from an input to an output.
Input: the number/ symbol put into a function.
Output: the number/ expression that comes out of a function.
Substitute: replace one variable with a number or new variable.
Inequality: an inequality compares who values showing if one is greater than, less than or equal to another
Linear: the difference between terms increases or decreases by the same value each time

Tier 3:

Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle
Quadratic: a polynomial with four terms (often simplified to three terms)

Two step function machines (algebra)

b → x 5 → 5b → + 4 → 5b + 4

÷ 5   - 4

IMPORTANT
Calculate the value at the end of each operation

c → + 4 → c+4 → x 5 → 5(c + 4) = 5c + 20

- 4   ÷ 5

NOTE:
The whole first output is multiplied by 5

Find functions from expressions

$\frac{f+5}{3}$

f add 5 then divide by 3

f → x 5 → + 3

NOTE: the difference in the two expressions

$\frac{f}{3} + 5$

f divided by then add 5

f → ÷ 3 → + 5

Sometimes it helps to try to explain the expression in word – and consider what has happened to the input

Substitution into an expression

2(x + 3)

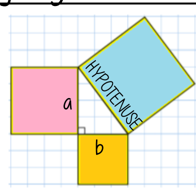
Add 3 to the input then times 2

Put the expression into a function machine

INPUT → + 3 → x 2 → OUTPUT

If x = 10
10 + 3 = 13.... 13 x 2 = 26

Pythagoras' theorem



Hypotenuse^2 = a^2 + b^2

This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

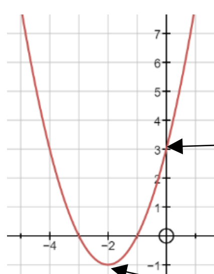
Places to look out for Pythagoras
- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Quadratic Graphs

$y = x^2 + 4x + 3$

If x^2 is the highest power in your equation then you have a quadratic graph.

It will have a parabola shape



Intersection with the y axis

Substitute the x values into the equation of your line to find the y coordinates

x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

Coordinate pairs for plotting $(-3, 0)$

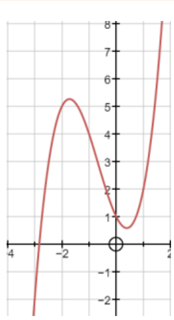
Plot all of the coordinate pairs and join the points with a curve (freehand)

Quadratic graphs are always symmetrical with the turning point in the middle

Interpret other graphs

Cubic Graphs

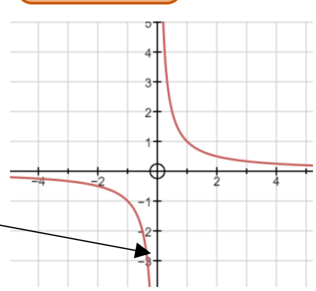
$y = x^3 + 2x^2 - 2x + 1$



If x^3 is the highest power in your equation then you have a cubic graph

Reciprocal Graphs

$y = \frac{1}{x}$

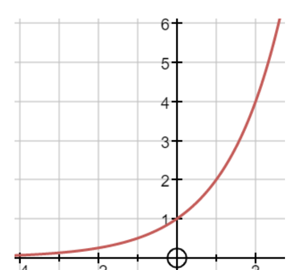


Reciprocal graphs never touch the y axis.
This is because x cannot be 0
This is an asymptote

Exponential Graphs

$y = 2^x$

Exponential graphs have a power of x



ALGEBRA... Functions and graphs(H)

What do I need to be able to do?

- Function notation
- Composite functions
- Inverse functions
- Quadratic and cubic graphs
- Reciprocal graphs
- Exponential graphs
- Trigonometric graphs (E)
- Recognise and sketch graphs
- Translation of a graph (E)
- Reflection of a graph (E)

Keywords

Tier 2:

Inverse: the operation that undoes what was done by the previous operation. (The opposite operation)

Function: a relationship that instructs how to get from an input to an output.

Input: the number/ symbol put into a function.

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Inequality: an inequality compares who values showing if one is greater than, less than or equal to another

Linear: the difference between terms increases or decreases by the same value each time

Tier 3:

Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle

Quadratic: a polynomial with four terms (often simplified to three terms)

Composite function: A combination of two or more functions to create a new function.

Identify and interpret roots, intercepts of quadratic functions graphically

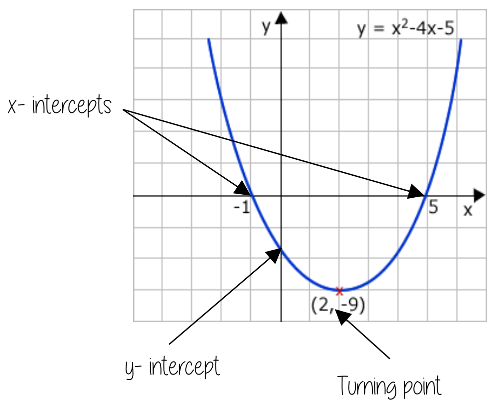
A root is a **solution**.

The roots of a quadratic are the **x-intercepts of the quadratic graph**

A **turning point** is the point where a quadratic turns.

On a **positive parabola**, the turning point is called a **minimum**.

On a **negative parabola**, the turning point is called a **maximum**.



Inverse Functions (H)

1. Write the function as $y = f(x)$
2. Rearrange to make x the subject.
3. Replace the y with x and the x with $f^{-1}(x)$

$f(x) = (1-2x)^5$. Find the inverse.

$$y = (1-2x)^5$$
$$\sqrt[5]{y} = 1-2x$$
$$1-\sqrt[5]{y} = 2x$$
$$\frac{1-\sqrt[5]{y}}{2} = x$$
$$f^{-1}(x) = \frac{1-\sqrt[5]{x}}{2}$$

Composite Functions (H)

fg(x) is the composite function that substitutes the function g(x) into the function f(x)

fg(x) means 'do g first, then f'

gf(x) means 'do f first, then g'

$f(x) = 5x-3$, $g(x) = \frac{1}{2}x+1$

What is $fg(4)$?

$$g(4) = \frac{1}{2}\times 4+1 = 3$$
$$f(3) = 5\times 3-3 = 12 = fg(4)$$

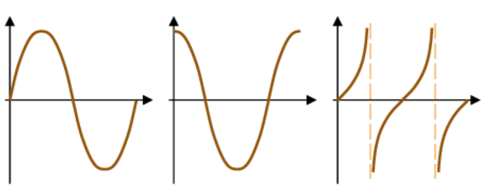
What is $fg(x)$?

$$fg(x) = 5\left(\frac{1}{2}x+1\right)-3 = \frac{5}{2}x+2$$

Trigonometric Graphs(H)

y= sin(x) and y= cos(x)
y is never more than 1 or less than -1

y= sin(x), y= cos(x) and y = tan(x)
Pattern repeats every 360°.



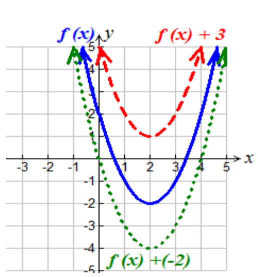
$y = \sin(x°)$ $y = \cos(x°)$ $y = \tan(x°)$

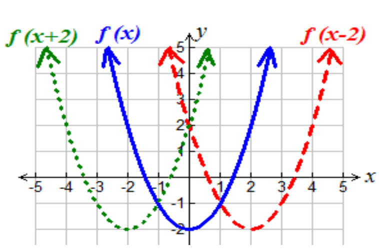
Translation of a graph(H)

Starting with the curve $y=f(x)$:

Translate $\binom{0}{a}$ for $y=f(x)+a$

Translate $\binom{-a}{0}$ for $y=f(x+a)$



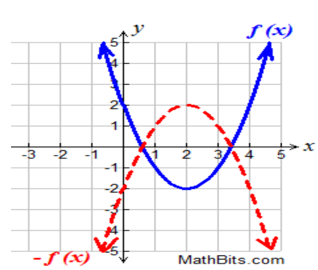


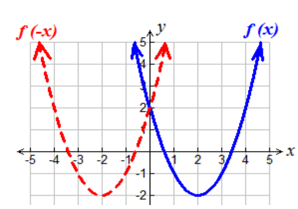
Reflection of a graph(H)

Starting with the curve $y=f(x)$:

Reflect in x axis for $y=-f(x)$

Reflect y axis for $y=f(-x)$





ALGEBRA… Equations and formulae

What do I need to be able to do?

- Solve equations
- Form and solve equations
- Form and solve equations with unknowns on both sides (E)
- Solve inequalities
- Form and solve inequalities (E)
- Form and solve more complex equations and inequalities (E)
- Solve simultaneous equations
- Solve simultaneous equations (one linear, one non-linear) (E)
- Quadratic equations
- Quadratic inequalities (E)
- Understand iterative processes
- Solve equations by iteration

Keywords

Tier 2:

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet.

Equation: an equation says that two things are equal – it will have an equals sign =

Expression: numbers, symbols and operators grouped together to show the value of something

Identity: On equation where both sides have variables that cause the same answer includes ≡

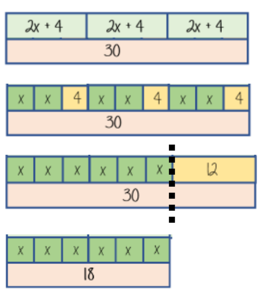
Inequality: an inequality compares two values showing if one is greater than, less than or equal to another

Formula: a fact or rule that uses mathematical symbols

Iteration: the act of repeating a process over and over again, often with the aim of approximating a desired result more closely.

Solve equations

$3(2x + 4) = 30$



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

-12 $\quad$ -12

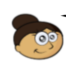
Substitute to check your answer. This could be negative or a fraction or decimal

$6x = 18$

$\div 6$ $\quad$ $\div 6$

$x = 3$

Form and solve inequalities



Two more than treble my number is greater than 11

Form

x → x3 → +2 → 11

$3x + 2 > 11$

Solve

x ← ÷3 ← −2 ← 11

$x > 3$

Equations with unknown on both sides

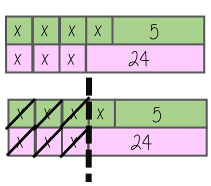
$4x + 5 = 3x + 24$

$-3x$ $\quad$ $-3x$

$x + 5 = 24$

-5 $\quad$ -5

$\underline{x = 19}$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$5(x + 4) < 3(x + 2)$

$5x + 20 < 3x + 6$

$2x + 20 < 6$

$2x < -14$

$\underline{x < -7}$

Check it!

$5(-8 + 4) < 3(-8 + 2)$

$5(-4) < 3(-6)$

$-20 < -18$ ✓

Rearranging Formulae (two step)

In a formula (make x the subject)

$xy - s = a$

$+s$ $\quad$ $+s$

$xy = a + s$

$\div y$ $\quad$ $\div y$

$x = \dfrac{a + s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g. Find the gradient of the line $2y - 4x = 9$

Make y the subject first $\quad y = \dfrac{4x + 9}{2}$

Gradient $= \dfrac{4}{2} = 2$

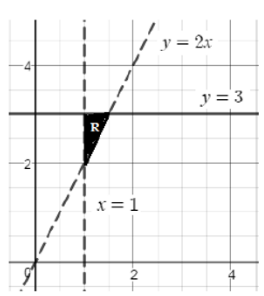
Linear Inequalities

If the inequality is strict ($x > 2$) then use a dotted line.

If the inequality is not strict ($x \leq 6$) then use a solid line.

Shade the region that satisfies:

$y > 2x, x > 1$ and $y \leq 3$



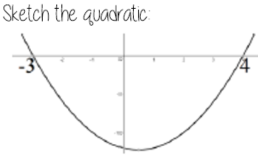
Quadratic Inequalities (H)

If the expression is > or ≥ then the answer will be above the x-axis.

If the expression is < or ≤ then the answer will be below the x-axis.

Solve the inequality $x^2 - x - 12 < 0$

Sketch the quadratic:



The required region is below the x-axis so, the final answer is:

$-3 < x < 4$

If the question had been > 0, the answer would have been:

$x < -3$ or $x > 4$

Rearranging Complex Formulae(H)

$s = vt - \frac{1}{2}at^2$ is a formula used in maths and physics.

Express a in terms of s, v and t

$$s = vt - \frac{1}{2}at^2$$

$+\frac{1}{2}at^2$ $\quad$ $s + \frac{1}{2}at^2 = vt$ $\quad$ $+\frac{1}{2}at^2$

$-s$ $\quad$ $\frac{1}{2}at^2 = vt - s$ $\quad$ $-s$

$\div t^2$ $\quad$ $\frac{1}{2}a = \frac{vt - s}{t^2}$ $\quad$ $\div t^2$

$\times 2$ $\quad$ $a = \frac{2(vt - s)}{t^2}$ $\quad$ $\times 2$

Iteration (H)

Approximate solutions to more complex equations can be found using a process called iteration. Iteration means repeatedly carrying out a process. To solve an equation using iteration, start with an initial value and substitute this into the iteration formula to obtain a new value, then use the new value for the next substitution, and so on.

To create an iterative formula, **rearrange** an equation with more than one x term to **make one of the x terms the subject**.

You will be given the first value to substitute in, often called $\boldsymbol{x_1}$.

Keep substituting in your previous answer until your answers are the same to a certain degree of accuracy. This is called converging to a limit.

Use the 'ANS' button on your calculator to keep substituting in the previous answer.

Use an iterative formula to find the positive root of $x^2 - 3x - 6 = 0$ to 3 decimal places.
$x_1 = 4$

Answer:

$$x^2 = 3x + 6$$
$$x = \sqrt{3x+6}$$
So $$x_{n+1} = \sqrt{3x_n + 6}$$

$x_1 = 4$
$x_2 = \sqrt{3 \times 4 + 6} = 4.242640\ldots$
$x_3 = \sqrt{3 \times 4.242640\ldots + 6}$
$= 4.357576\ldots$

Keep repeating…

$x_7 = 4.372068.. = 4.372\ (3dp)$
$x_8 = 4.372208.. = 4.372\ (3dp)$

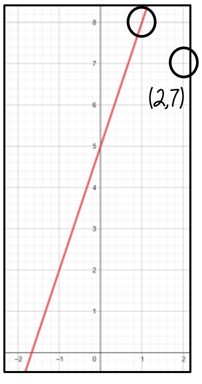
So answer is $x = 4.372\ (3dp)$

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

Does the coordinate (1,8) lie on the line y=3x+5?

This coordinate represents x=1 and y=8



$y = 3x + 5$
$8 = 3(1) + 5$

As the substitution makes the equation correct the coordinate (1,8) IS on the line y=3x+5

Is (2,7) on the same line?

$7 \neq 3(2) + 5$

No 7 does NOT equal 6+5

Substituting known variables

A line has the equation 3x + y = 14

Two different variables, two solutions

Stephanie knows the point x = 4 lies on that line. Find the value for y

$3x + y = 14$ | x | x | x | y | 14 |

$x = 4$

$3(4) + y = 14$ | 4 | 4 | 4 | y | 14 |

$12 + y = 14$
$-12 \quad -12$
$y = 2$ | y | 2 |

Substituting in an expression

x = 2y | y | y | x |

x + y = 30 | x | y | 30 |

Pair of simultaneous equations (two representations)

Substitute 2y in place of the x variable as they represent the same value

$x = 2y$ $x + y = 30$

$3y = 30$ | y | y | y | 30 |

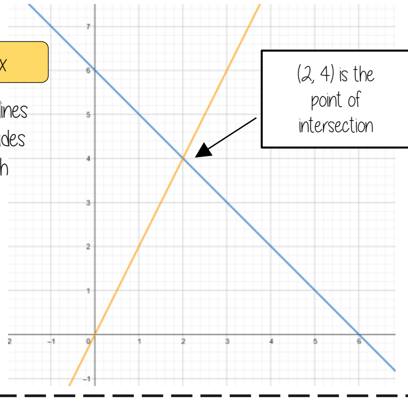
$3y = 30$
$\div 3 \quad \div 3$
$y = 10$ | y | 10 |

$x = 2y$ | 10 | 10 | x |
$x = 20$

Solve graphically

x + y = 6   y = 2x

Linear equations are straight lines. The point of intersection provides the x and y solution for both equations



(2, 4) is the point of intersection

The solution that satisfies both equations is

$x = 2$ and $y = 4$

Solve by subtraction

18	x x x y y
10	x y y
8	x x

$x = 4$
$y = 3$

$3x + 2y = 18$
$-\quad x + 2y = 10$

$2x = 8$
$\div 2 \quad \div 2$
$x = 4$

$x + 2y = 10$
$(4) + 2y = 10$
$-4 \quad -4$
$2y = 6$
$\div 2 \quad \div 2$
$y = 3$

x x x y y = 18
x y y = 10

x x ~~x~~ ~~y~~ ~~y~~ = 18
~~x~~ ~~y~~ ~~y~~ = 10

x x = 8
x = 4
y = 3

Solve by addition

Addition makes zero pairs

$3x + 2y = 16$
$+\ 6x - 2y = 2$

$9x = 18$
$\div 9 \quad \div 9$
$x = 2$

$3x + 2y = 16$
$3(2) + 2(y) = 16$
$6 + 2y = 16$
$-6 \quad -6$
$2y = 10$
$y = 5$

x x x y y = 16
x x x -y -y
x x x = 2

x x x x x x x x x = 18

x = 2
y = 5

Solve by adjusting one

$h + j = 12$
$2h + 2j = 29$

No equivalent values

12: | h | j |
29: | h | h | j | j | j |

$2h + 2j = 24$
$2h + 2j = 29$

24: | h | h | j | j |
29: | h | h | j | j | j |

By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

Solve by adjusting both

$2x + 3y = 39$
$5x - 2y = -7$

x x y y y = 39
x x x -y -y x x = −7

Use LCM to make equivalent x OR y values. Because of the negative values using zero pairs and y values is chosen choice

$4x + 6y = 78$
$15x - 6y = -21$

= 78
= −21

Now solve by addition

Addition makes zero pairs

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

Does the coordinate (-3, 9) lie on the curve y = x^2 and the straight line y=2x+15?

This coordinate represents x=-3 and y=9

$y = x^2$

$y = (-3)^2$

$\boldsymbol{y = 9}$

$y = 2x + 15$

$y = 2(-3) + 15$

$y = -6 + 15$

$\boldsymbol{y = 9}$

As the substitution makes the equation correct the coordinate (-3, 9) IS on the curve y = x^2 and the straight line y=2x+15

Solve graphically: linear/quadratic(H)

y = x^2

y = x + 2

The points of intersection provide the x and y solutions for both equations

Intersection at $(2, 4)$

$\boldsymbol{x = 2}$ $\boldsymbol{y = 4}$

Intersection at $(-1, 1)$

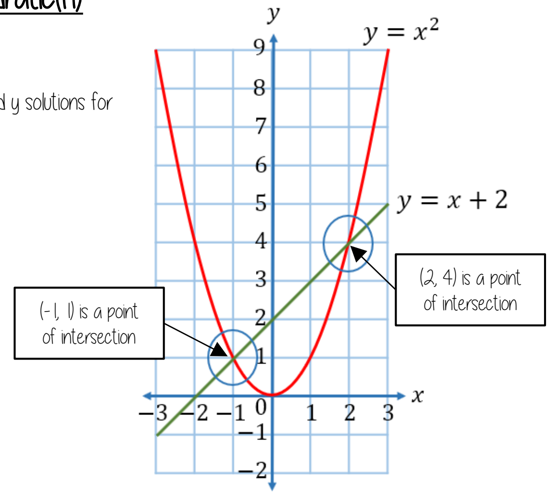
$\boldsymbol{x = -1}$ $\boldsymbol{y = 1}$

The solution that satisfies both equations is

$x = 2 , y = 4$

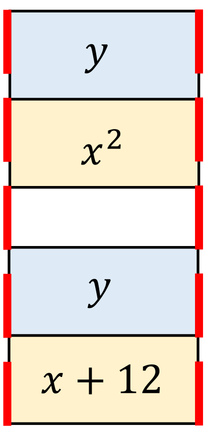
and

$x = -1 , y = 1$



Solve by rearranging: linear/quadratic(H)

If $y = x^2$ and $y = x + 12$ then $x^2 = x + 12$



$x^2 = x + 12$

Rearrange to form a quadratic equation equal to zero

$x^2 - x - 12 = 0$

Factorise and solve the resulting linear equations

$(x - 4)(x + 3) = 0$

$\boldsymbol{x = 4 , x = -3}$

Substitute the values of x into one of the original equations to find the corresponding values for y

$y = x^2$

$y = (4)^2$

$\boldsymbol{y = 16}$

$y = (-3)^2$

$\boldsymbol{y = 9}$

The solutions that satisfy both equations are

$\boldsymbol{x = 4}$ and $\boldsymbol{y = 16}$

$\boldsymbol{x = -3}$ and $\boldsymbol{y = 9}$

Solve by substitution: linear/quadratic(H)

Determine the variable to substitute

$y^2 + x^2 = 16$

$y = x + 4$

Substitute, expand and rearrange to form a quadratic equation equal to zero

$(x + 4)^2 + x^2 = 16$

$x^2 + 8x + 16 + x^2 = 16$

$2x^2 + 8x = 0$

Factorise and solve the resulting linear equations

$2x(x + 4) = 0$

$\boldsymbol{x = 0}$ $\boldsymbol{x = -4}$

Substitute these values of x into one of the original equations to find the corresponding values for y

$y = x + 4$

$y = 0 + 4$

$\boldsymbol{y = 4}$

$y = -4 + 4$

$\boldsymbol{y = 0}$

The solutions that satisfy both equations are

$\boldsymbol{x = 0}$ and $\boldsymbol{y = 4}$

$\boldsymbol{x = -4}$ and $\boldsymbol{y = 0}$

PROPORTION... Rates

What do I need to be able to do?

- Convert between hours and minutes
- Speed, distance and time
- Interpret distance-time graphs
- Draw distance-time graphs
- Velocity-time graphs
- Density and pressure
- Convert compound units (E)

Keywords

Tier 2:

Convert: change

Mass: a measure of how much matter is in an object. Commonly measured by weight.

Origin: the coordinate (0, 0)

Volume: the amount of 3D space a shape takes up

Substitute: putting numbers where letters are – replacing numbers into a formula

Speed, Distance, Time

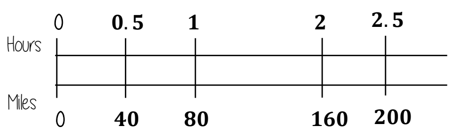
"per" for every

e.g. 80 miles per hour (mph)

Travel 80 miles every hour

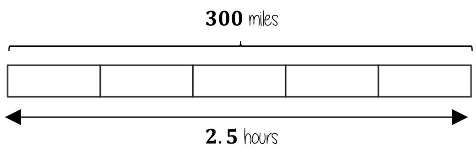
$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

You can use a double number line to help you calculate distance



e.g. A boat travels at a constant speed for **2.5** hours. It travels 300 miles.

Bar models can help to calculate mph



Each part is half an hour

Each part is 60 miles

Speed, Distance, Time

minutes $\xrightarrow{\div 60}$ hours

hours $\xrightarrow{\times 60}$ minutes

Before calculations – make sure you are working in the same units as the speed

Learn or learn how to rearrange the formular for speed, distance and time

Substitute in the variables given

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

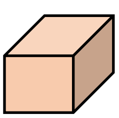
$$\text{distance} = \text{speed} \times \text{time}$$

Density, Mass, Volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume} = \frac{\text{mass}}{\text{density}}$$

$$\text{mass} = \text{volume} \times \text{density}$$



volume of prism = Area of cross section × Depth

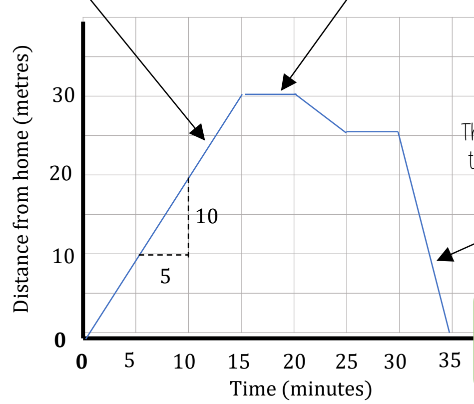
Distance – Time graphs

$$\text{Gradient} = \text{speed}$$

The steeper a gradient the faster the speed

$\frac{10}{5}$ = 2 metres per min

Horizontal lines represent staying still



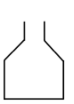
The distance coming closer to home shows the return journey

Units are important. Meters per minute

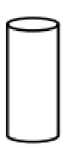
Velocity – Time graphs

Velocity-time graphs show velocity (y-axis) against time (x-axis); the gradient (change in velocity ÷ time) equals acceleration, while the area under the graph (using shapes like triangles, rectangles, trapeziums) represents distance. A horizontal line means constant velocity (zero acceleration), a sloping line means constant acceleration, and a curved line indicates variable acceleration, requiring tangents for instantaneous acceleration.

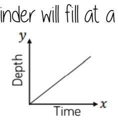
Flow problems & graphs



This will fill at a constant rate, then as the space decreases it will speed up and the neck of the bottle fill at a faster constant speed



The cylinder will fill at a constant speed



Units are important. Ensure any volume calculations are the same unit as the rate of flow

Rates of change & units

Common rates of change relationships

Revisit your conversions between units of length and capacity

Speed: miles **per** hour

Exchange rates: euros **per** pounds

Density: mass **per** volume

metres $\xrightarrow{\div 1000}$ kilometres

kilometres $\xrightarrow{\times 1000}$ metres

GEOMETRY… Angles, bearings and trigonometry

What do I need to be able to do?

- Angles in lines and shapes
- Solve problems with angles in parallel lines
- Solve problems with angles and algebra (E)
- Compass points and related angles
- Understand and represent bearings
- Measure and read bearings
- Scale drawings using bearings
- Calculate bearings using angles rules (E)
- Choose appropriate methods to solve problems with right-angled triangles
- Solve bearings problems using Pythagoras and trigonometry (E)
- Sine and cosine rules
- Solve bearings problems using the sine and cosine rules (E)

Keywords

Tier 2:

Angle: the amount of turn between two lines around their common point

Bearing: the angle in degrees measured clockwise from North.

Perpendicular: where two lines meet at 90°

Parallel: straight lines always the same distance apart and never touch. They have the same gradient.

Clockwise: moving in the direction of the hands on a clock.

Construct: to draw accurately using a compass, protractor and or ruler or straight edge.

Scale: the ratio of the length of a drawing to the length of the real thing

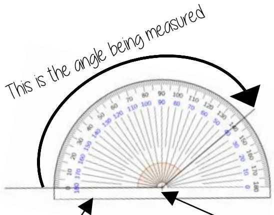
Protractor: an instrument used in measuring or drawing angles.

Tier 3:

Cardinal directions: the directions of North, South, East, West

Measure angles to 180°

This is the angle being measured



Read from 0° on the base line. Remember to use estimation. This is an obtuse angle so between 90 ° and 180 °

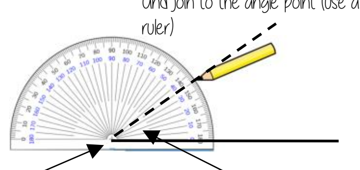
The base line follows the line segment

Make sure the cross is at the point the two lines meet

Draw angles up to 180°

Draw a 35° angle

Make a mark at 35° with a pencil And join to the angle point (use a ruler)



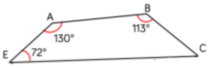
Make sure the cross is at the end of the line (where you want the angle)

The angle

Angle notation

The letter in the middle is the angle

The arc represents the part of the angle



Angle Notation: three letters $\hat{ABC}$ This is the angle at B = 113 °

$\angle ABC$ is also used to represent the angle at B.

Scale drawings

1 : 20

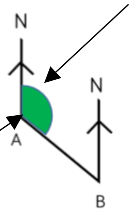
For every 1cm on the model there are 20cm in real life

Remember: Scale drawings ONLY change lengths and distances. Angles remain the same

Understand and represent bearings

- a bearing is always measured from NORTH
- It is always given as three figures

The angle indicated starts from the North line at a and joins the path connecting a to B.



This angle shows the bearing of **B from A**

The bearing of B from a is calculated by measuring the highlighted angle

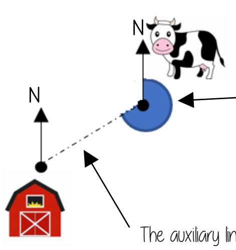
The sentence… "Bearing of ____ from _____" is really important in identifying the bearing being represented

Using **estimation** it is clear this angle is between 090° and 180°

Directions

Clockwise Anti-Clockwise

Measure and read bearings



The bearing of the cow to the barn

This angle is measured from NORTH

It is measured in a clockwise direction

Estimation indicates this angle is between 180° and 270°

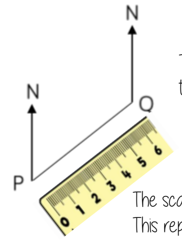
Use a protractor to measure accurately

Remember: bearings are written as three figures.

The auxiliary line is drawn to help you measure and draw the angle that is measured to represent the bearing

Scale drawings using bearings

Remember – angles DO NOT change size in scaled drawings



The bearing measurements do not change from "real life" to images

The units in the ratio scale are the same

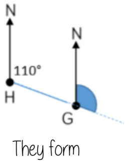
The scale may need to be calculated from the image. This represents 30km from P to Q

6cm = 30km

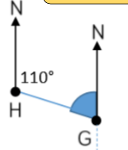
6:3,000,000

Bearings with angle rules

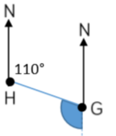
Because two North lines are PARALLEL….



They form **corresponding** angles and therefore are the same size



They form **co-interior** angles and add up to 180°

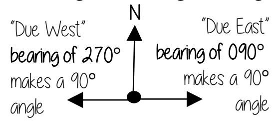


They form **alternate** angles and therefore are the same size

Bearings with right-angled geometry

Look for Right-angles: Pythagoras Trigonometry (Sin, Cos, Tan)

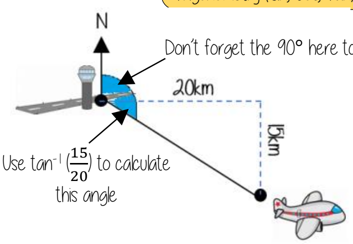
"Due West" **bearing of 270°** makes a 90° angle



"Due East" **bearing of 090°** makes a 90° angle

A plane flies East for 20km then turns South for 15km. Find the bearing of the plane from where it took off.

Don't forget the 90° here too

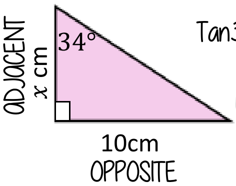


Use $\tan^{-1}\left(\frac{15}{20}\right)$ to calculate this angle

Tangent ratio: side lengths

$$\text{Tan}\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Substitute the values into the tangent formula



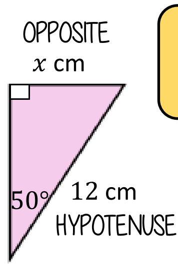
$\text{Tan}34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times \text{Tan}34 = 10$

$x = \frac{10}{\text{Tan}34} = 14.8\text{cm}$

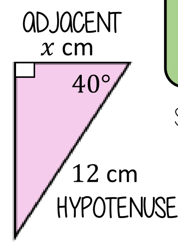
Sin and Cos ratio: side lengths



$$\text{Sin}\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$$

NOTE

The Sin(x) ratio is the same as the Cos(90-x) ratio

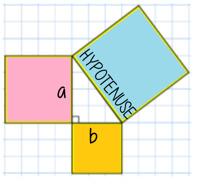


$$\text{Cos}\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$$

Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem



$\text{Hypotenuse}^2 = a^2 + b^2$

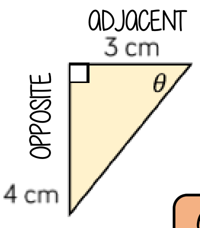
This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras

- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$$\theta = \text{Tan}^{-1}\frac{\text{opposite side}}{\text{adjacent side}}$$

$$\theta = \text{Sin}^{-1}\frac{\text{opposite side}}{\text{hypotenuse side}}$$

$$\theta = \text{Cos}^{-1}\frac{\text{adjacent side}}{\text{hypotenuse side}}$$

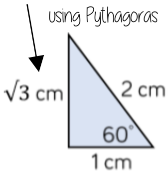
$\text{Tan}\theta = \frac{3}{4}$

$\theta = \text{Tan}^{-1}\frac{3}{4}$

$\theta = 36.9°$

Key angles

This side could be calculated using Pythagoras

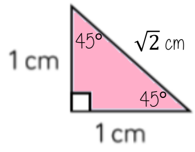


Because trig ratios remain the same for similar shapes you can generalise from the following statements

$\text{Tan}30 = \frac{1}{\sqrt{3}}$ $\text{Tan}60 = \sqrt{3}$

$\text{Cos}30 = \frac{\sqrt{3}}{2}$ $\text{Cos}60 = \frac{1}{2}$

$\text{Sin}30 = \frac{1}{2}$ $\text{Sin}60 = \frac{\sqrt{3}}{2}$



$\text{Tan}45 = 1$

$\text{Cos}45 = \frac{1}{\sqrt{2}}$

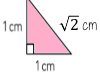
$\text{Sin}45 = \frac{1}{\sqrt{2}}$

Key angles 0° and 90°

$\text{Tan}0 = 0$

$\cancel{\text{Tan}90}$

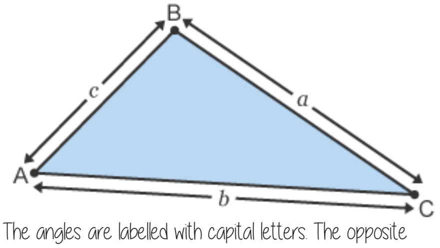
This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle



$\text{Sin}0 = 0$ $\text{Sin}90 = 1$

$\text{Cos}0 = 1$ $\text{Cos}90 = 0$

The Sine Rule (H)



The angles are labelled with capital letters. The opposite sides are labelled with lower case letters.

Notice that an angle and its opposite side are the same letter.

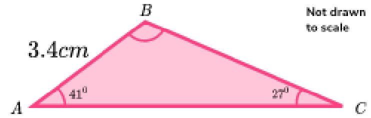
The sine rule is: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

This version is used to calculate lengths.

It can be rearranged to: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

This version is used to calculate angles.

Calculate the length BC. Write your answer to two decimal places.



1. Label each angle (A, B, C) and each side (a, b, c) of the triangle.
2. State the sine rule then substitute the given values into the equation
3. Solve the equation.

The Cosine Rule (H)

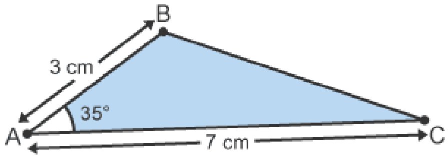
The cosine rule is: $a^2 = b^2 + c^2 - 2bc\cos A$

This version is used to calculate lengths.

It can be rearranged to: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

This version is used to calculate angles.

Calculate the length BC. Give the answer to three significant figures.



Use the form $a^2 = b^2 + c^2 - 2bc\cos A$ to calculate the length.

$\text{BC}^2 = 3^2 + 7^2 - 2 \times 3 \times 7\cos 35$

$\text{BC}^2 = 23.59561414\ldots$

BC = 4.86 cm

GEOMETRY...

Constructions & loci

What do I need to be able to do?

- Construct triangles
- Construct polygons
- Construct an angle bisector and a perpendicular bisector
- Construct a perpendicular from or to a point
- Locus of distance from a point and a straight line
- Locus equidistant from two points and perpendicular bisectors
- Locus equidistant from two lines and angle bisectors
- Solve problems with loci

Keywords

Tier 2:

Locus: set of points with a common property

Equidistant: the same distance

Perpendicular: lines that meet at 90°

Arc: part of a curve

Bisector: a line that divides something into two equal parts

Congruent: the same shape and size

Tier 3:

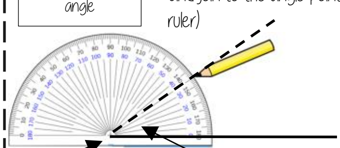
Protractor: piece of equipment used to measure and draw angles

Discorectangle: (a stadium) – a rectangle with semi circles at either end

Draw and measure angles

Draw a 35° angle

Make a mark at 35° with a pencil And join to the angle point (use a ruler)



The angle

Make sure the cross is at the end of the line (where you want the angle)

Scale drawings

A picture of a car is drawn with a scale of 1:30

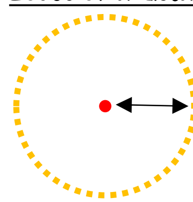
For every 1cm on my image is 30cm in real life

The car image is 10cm

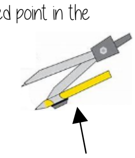
Image	:	Real life
1cm	:	30cm
10cm	:	300cm

× 10   × 10

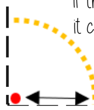
Locus of a distance from a point



All points are equidistant (the same distance) from the fixed point in the middle.



If the point is in the corner it can only make a quarter circle



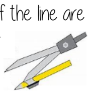
Equipment needed

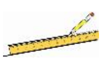
The radius is the distance from the fixed point

Locus of a distance from a straight line

All points are equidistant (the same distance) from line

The ends of the line are fixed points

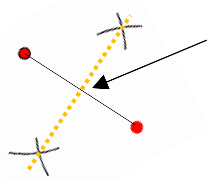




Equipment needed

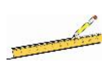
The line is straight so a ruler is used for the straight lines parallel to your original line

Locus equidistant from two points



Also a perpendicular bisector

Because if the points are joined, this new line intersects it at a 90°

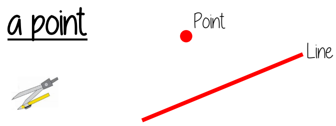


Keep the compass the same size and draw two arcs from each point

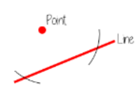
Join the intersections with a ruler.

All points on this line are equidistant from both points

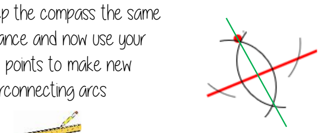
Construct a perpendicular from a point



Use a compass and draw an arc that cuts the line. Use the point to place the compass



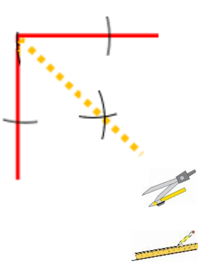
Keep the compass the same distance and now use your new points to make new interconnecting arcs



Connecting the arcs makes the bisector

If P is a point on the line the steps are the same

Locus of a distance from two lines



Also an angle bisector

This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle

Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle

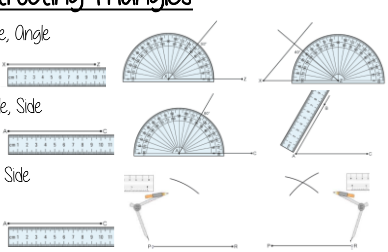
Join the vertex to the intersection

Constructing Triangles

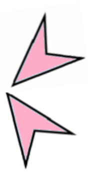
Side, Angle, Angle

Side, Angle, Side

Side, Side, Side

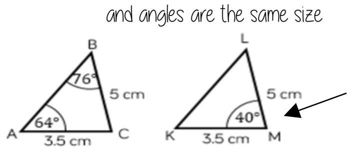


Congruent figures



Congruent figures are identical in size and shape – they can be reflections or rotations of each other

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**

Congruent triangles

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

GEOMETRY…

Transformations

What do I need to be able to do?

- Enlargement (positive integer scale factor)
- Enlargement (fractional scale factor)
- Enlargement (negative scale factor)
- Reflection
- Rotation
- Translation
- Describe a transformation
- Find the result of a series of transformations
- Invariant points and lines (E)

Keywords

Tier 2:

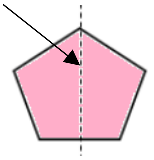
Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Similar: when one shape can become another with a reflection, rotation, enlargement or translation.
Reflect: mapping of one object from one position to another of equal distance from a given line.
Rotate: a rotation is a circular movement.
Vertex: a point where two or more-line segments meet.
Perpendicular: lines that cross at 90°
Symmetry: when two or more parts are identical after a transformation.
Regular: a regular shape has angles and sides of equal lengths.
Invariant: a point that does not move after a transformation.

Tier 3:

Scale Factor: the multiplier of enlargement
Centre of enlargement: the point the shape is enlarged from

Lines of symmetry

Mirror line (line of reflection)

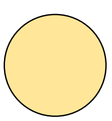


Rhombus
two lines of symmetry

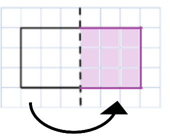
Parallelogram
No lines of symmetry

Shapes can have more than one line of symmetry… This regular polygon (a regular pentagon has 5 lines of symmetry)

A circle has an infinite amount of lines of symmetry

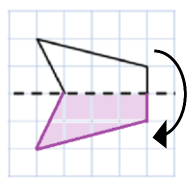


Reflect horizontally/ vertically (1)



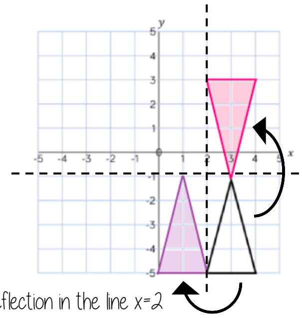
Note: a reflection doubles the area of the original shape

Reflection in a vertical line



Reflection in a horizontal line

Reflection on an axis grid

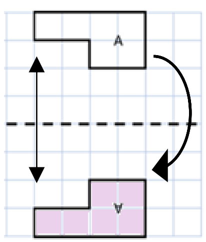


Reflection in the line y=-2

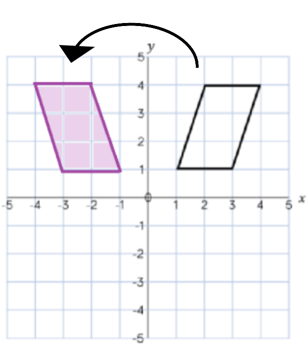
Reflection in the line x=2

Reflect horizontally/ vertically (2)

All points need to be the same distance away from the line of reflection



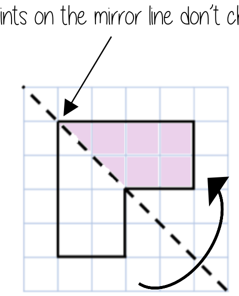
Reflection in the line y axis – this is also a reflection in the line x=0



Lines parallel to the x and y axis
REMEMBER
Lines parallel to the x-axis are y = ____
Lines parallel to the y-axis are x = ____

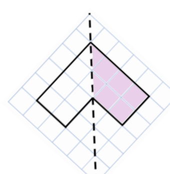
Reflect Diagonally (1)

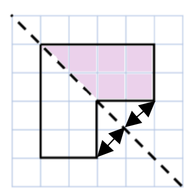
Points on the mirror line don't change position



Fold along the line of symmetry to check the direction of the reflection

Turn your image
If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)

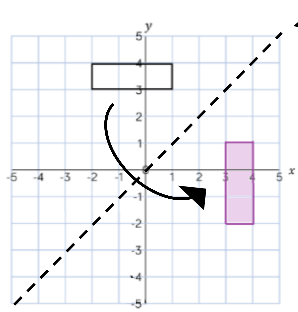


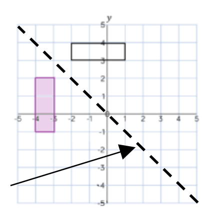


Perpendicular lines to and from the mirror line can help you to plot diagonal reflections

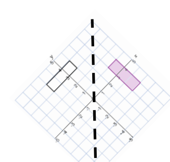
Reflect Diagonally (2)

This is the line y = x (every y coordinate is the same as the x coordinate along this line)





This is the line y = - x
The x and y coordinate have the same value but opposite sign



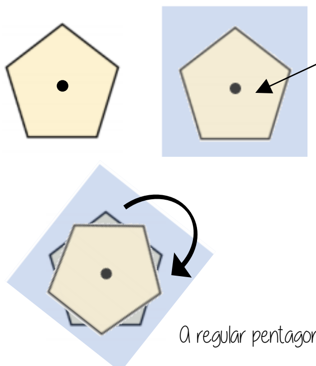
Turn your image
If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)

GEOMETRY...

Transformations

Rotational Symmetry

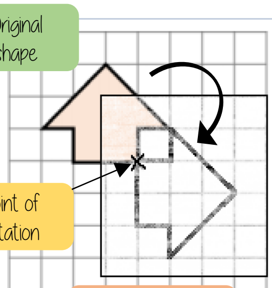
Tracing paper helps check rotational symmetry



1. Trace your shape (mark the centre point)
2. Rotate your tracing paper on top of the original through 360°
3. Count the times it fits back into itself

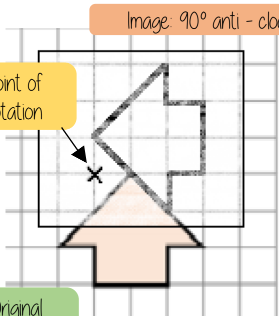
a regular pentagon has rotational symmetry of order 5

Rotate from a point (in a shape)



1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

Rotate from a point (outside a shape)



1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

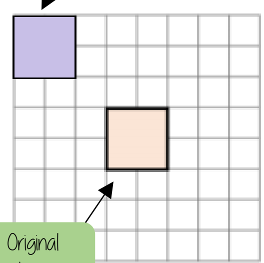
Translation and vector notation

Vector Notation → $\begin{pmatrix}1\\-2\end{pmatrix}$

How far left or right to move
Negative value (left)
Positive value (right)

How far up or down to move
Negative value (down)
Positive value (up)

Translation $\begin{pmatrix}-3\\3\end{pmatrix}$

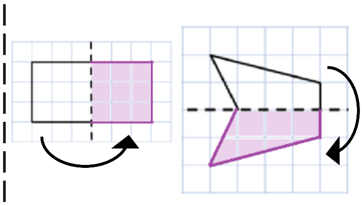


Every vertex has been translated by the same amount

$\begin{pmatrix}-3\\3\end{pmatrix}$

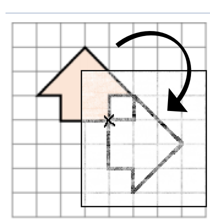
The image has been moved 3 squares to the left and 3 squares up

Compare rotations and reflections



Reflections are a mirror image of the original shape.

Information needed to perform a reflection:
- Line of reflection (Mirror line)



Rotations are the movement of a shape in a circular motion

Information needed to perform a rotation:
- Point of rotation
- Direction of rotation
- Degrees of rotation

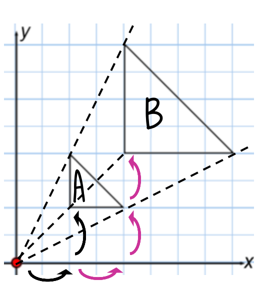
Positive scale factors

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

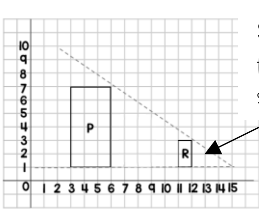
The distance from the point enlarges by 2



Fractional scale factors

Fractions less than 1 make a shape SMALLER

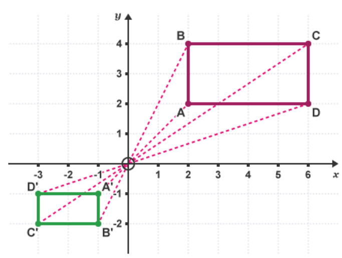
R is an enlargement of P by a scale factor $\frac{1}{3}$ from centre of enlargement (15,1)



SF: $\frac{1}{3}$ - R is three times smaller than P

Negative scale factors(H)

An enlargement with a negative scale factor produces an image on the other side of the centre of enlargement. The image appears upside down.



The rectangle ABCD has been enlarged by a scale factor of $-\frac{1}{2}$